Blue Print (As per PU Board)

Topic	1 mark questions	2 marks questions	3 marks questions	5 marks questions	6 marks questions	Total Marks
Trigonometric Functions	1	2	1	1	1	19

One mark questions

1. **What do you mean by measure of an angle?**
   Answer: The amount of rotation from initial position to terminal position is called measure of an angle.
2. **Prove that** $\cot A \cdot \sec A \cdot \sin A = 1$
   Answer: LHS $= \frac{\cos A}{\sin A} \cdot \frac{1}{\cos A} \cdot \sin A = 1 = \text{RHS}$
3. **Define one radian.**
   Answer: A Radian is the angle subtended at the centre of a circle by an arc, whose length is equal to its radius. 1 radian is denoted by 1^c. One radian is also called one circular measure.

Two marks questions

4. **With usual notations prove that** $l = r\theta$**.**
   Answer: We know that in a circle of radius r, an arc of length r, subtends an angle of one radian at the center of the circle. Since the angle subtended at the center by an arc of length r is one radian, the angle subtended by an arc of length l has the measure $= \frac{l}{r}$. Thus if θ is the angle subtended at the center by an arc of length l then $\frac{l}{r} = \theta \therefore l = r\theta$.
5. **With usual notations prove that the area of a sector of a circle is given by** $\frac{1}{2}r^2\theta$ **or** $\frac{1}{2}rl$
   Answer: We know that in a circle of radius r, the angle 2π radian trances the area πr^2. Therefore the area of the sector tracing an angle θ radian at the center is $A = \frac{\pi r^2}{2\pi} \times \theta = \frac{1}{2}r^2\theta$.
   Since $l = r\theta$ we get $A = \frac{1}{2}r^2\theta = \frac{1}{2}r.r\theta = \frac{1}{2}rl$
6. **Prove that** $\text{cosec}^2 A \cdot \tan^2 A - 1 = \tan^2 A$
   Answer: LHS $= \left(1 + \cot^2 A\right) \cdot \tan^2 A - 1$
   $= \tan^2 A + \cot^2 A \cdot \tan^2 A - 1$
   $= \tan^2 A + (\cot A \cdot \tan A)^2 - 1$
   $= \tan^2 A + 1 - 1 = \tan^2 A = \text{RHS}$

Three marks questions

7. **The angles of a triangle are in the ratio** $4:5:6$**. Find them in radian and degree**
   Answer: Let A, B and C be the angles of the triangle.
   Now $A:B:C = 4:5:6 \Rightarrow A = 4\theta, B = 5\theta, C = 6\theta$
   $A + B + C = \pi \Rightarrow 4\theta + 5\theta + 6\theta = \pi \Rightarrow 15\theta = \pi \therefore \theta = \frac{\pi}{15}$
   $\therefore A = 4\theta = 4 \times \frac{\pi}{15} = \frac{4\pi}{15}, B = 5\theta = 5 \times \frac{\pi}{15} = \frac{\pi}{3}, C = 6\theta = 6 \times \frac{\pi}{15} = \frac{2\pi}{3}$

$A+B+C=180°\Rightarrow 15\theta=180°\ \therefore\theta=\frac{180°}{15}=12°$

$\therefore A=4\theta=4\times12°=48°,\ B=5\times12°=60°,\ C=6\theta=6\times12°=72°$

8. **Prove that** $(1-\sin A+\cos A)^2=2(1-\sin A)(1+\cos A)$

Answer: $LHS=\left[(1-\sin A)+\cos A\right]^2$

$=(1-\sin A)^2+\cos^2 A+2(1-\sin A)\cdot\cos A$

$=(1-\sin A)^2+\left(1-\sin^2 A\right)+2(1-\sin A)\cdot\cos A$

$=(1-\sin A)^2+(1-\sin A)\cdot(1+\sin A)+2(1-\sin A)\cdot\cos A$

$=(1-\sin A)[1-\sin A+1+\sin A+2\cos A]$

$=(1-\sin A)[2+2\cos A]=2(1-\sin A)(1+\cos A)=RHS$

9. **Prove that** $\frac{\tan^2 60°-2\tan^2 45°}{3\sin^2 45°\cdot\sin 90°+\cos^2 60°\cdot\cos^2 0°}=\frac{4}{7}$

Answer: $LHS=\frac{\left(\sqrt{3}\right)^2-2(1)^2}{3\left(\frac{1}{\sqrt{2}}\right)^2\cdot1+\left(\frac{1}{2}\right)^2\cdot(1)^2}=\frac{3-2}{\frac{3}{2}+\frac{1}{4}}=\frac{1}{\frac{7}{4}}=\frac{4}{7}=\text{RHS}$

Five marks questions

10. **Prove that** $\cos 20°\cos 40°\cos 60°\cos 80°=\frac{1}{16}.$

Answer: $LHS=\frac{1}{2}\left[\cos(20°+40°)+\cos(20°-40°)\right]\frac{1}{2}\cdot\cos 80°$

$=\frac{1}{4}\left[\cos 60°+\cos(-20°)\right]\cdot\cos 80°$

$=\frac{1}{4}\left[\frac{1}{2}+\cos 20°\right]\cdot\cos 80°=\frac{1}{4}\left[\frac{1+2\cos 20°}{2}\right]\cdot\cos 80°$

$=\frac{1}{8}[\cos 80°+2\cos 80°\cos 20°]$

$=\frac{1}{8}\left[\cos 80°+\cos(80°+20°)+\cos(80°-20°)\right]$

$=\frac{1}{8}[\cos 80°+\cos 100°+\cos 60°]$

$=\frac{1}{8}\left[\cos 80°+\cos(180°-80°)+\frac{1}{2}\right]$

$=\frac{1}{8}\left[\cos 80°-\cos 80°+\frac{1}{2}\right]=\frac{1}{16}$

11. **Prove that** $(\cos x+\cos y)^2+(\sin x-\sin y)^2=4\cos^2\left(\frac{x+y}{2}\right)$.

Answer: $(\cos x+\cos y)^2+(\sin x-\sin y)^2$

$=\cos^2 x+\cos^2 y+2\cos x.\cos y+\sin^2 x+\sin^2 y-2\sin x.\sin y$

$=\cos^2 x+\sin^2 x+\cos^2 y+\sin^2 y+2(\cos x.\cos y-\sin x.\sin y)$

$= 1 + 2 + 2\cos(x+y)$

$= 2 + 2\cos(x+y) = 2\{1+\cos(x+y)\} = 2.2\cos^2\left(\frac{x+y}{2}\right)$

$= 4\cos^2\left(\frac{x+y}{2}\right).$

12. **Prove that** $\sin^2 x + \sin^2\left(x+\frac{2\pi}{3}\right) + \sin^2\left(x-\frac{2\pi}{3}\right) = \frac{3}{2}$**.**

Answer: $\sin^2 x + \sin^2\left(x+\frac{2\pi}{3}\right) + \sin^2\left(x-\frac{2\pi}{3}\right)$

$= \frac{1-\cos 2x}{2} + \frac{1-\cos 2\left(x+\frac{2\pi}{3}\right)}{2} + \frac{1-\cos 2\left(x-\frac{\pi}{3}\right)}{2}$

$= \frac{1-\cos 2x}{2} + \frac{1-\cos\left(2x+\frac{\pi}{3}\right)}{2} + \frac{1-\cos\left(2x-\frac{4\pi}{3}\right)}{2}$

$= \frac{1-\cos 2x + 1 - \cos\left(2x+\frac{4\pi}{3}\right) + 1 - \cos\left(2x-\frac{4\pi}{3}\right)}{2}$

$= \frac{3-\left\{\cos 2x + \cos\left(2x+\frac{4\pi}{3}\right) + \cos\left(2x-\frac{4\pi}{3}\right)\right\}}{2}$

$\frac{3-\left\{\cos 2x + 2\cos 2x \cdot \cos\frac{4\pi}{3}\right\}}{2}$

$\left\{\cos\frac{4\pi}{3} = \cos\frac{3\pi+\pi}{3} = \cos\left(\pi+\frac{\pi}{3}\right) = -\cos\frac{\pi}{3} = -\frac{1}{2}\right\}$

$= \frac{3-\left\{\cos 2x + 2\cos 2x \cdot \left(-\frac{1}{2}\right)\right\}}{2} = \frac{3-\{\cos 2x - \cos 2x\}}{2} = \frac{3}{2}.$