**Blue Print (As per PU Board)**

| Topic | 1 mark questions | 2 marks questions | 3 marks questions | 5 marks questions | Total Marks |
|---|---|---|---|---|---|
| Principle of Mathematical Induction | – | – | – | 1 | 5 |

**Questions and Answers**

1. $1+2+3+....+n=\frac{n(n+1)}{2}$

Answer: Let $P(n):1+2+3+........+n=\frac{n(n+1)}{2}$

For $n=1,\ P(1):1=\frac{1(1+1)}{2}=\frac{2}{2}=1$, which is true.

Assuming that $P(k)$ is true for some +ve integer $k$, we have,

$1+2+3+..........+k=\frac{k(k+1)}{2}-------(1)$

We shall now prove that $P(k+1)$ is also true, now we have

$1+2+3+........+k+(k+1)=\frac{k(k+1)}{2}+(k+1)$ (using (1))

$=(k+1)\left(\frac{k}{2}+1\right)=\frac{(k+1)(k+2)}{2}=\frac{(k+1)(k+1+1)}{2}$

Thus $P(k+1)$ is true whenever $P(k)$ it true.

Hence from principal of Mathematical Induction (PMI) the statement $P(n)$ is true for all natural numbers $n$.

2. $1^2+2^2+3^2+...........+n^2=\frac{n(n+1)(2n+1)}{6}$

Answer: Let $P(n):1^2+2^2+3^2+.........+n^2=\frac{n(n+1)(2n+1)}{6}$

For $n=1,\ P(1):1^2=\frac{1(1+1)(2.1+1)}{6}=\frac{1(2)(3)}{6}=1\Rightarrow 1=1$, which is true.

Assume that $P(k)$ is true for some positive integer $k$, we have,

$1^2+2^2+3^2+........+k^2=\frac{k(k+1)(2k+1)}{6}$ ............(1)

Now we shall prove that $P(k+1)$ is also true, now we have,

$1^2+2^2+3^2+.....+k^2+(k+1)^2=\frac{k(k+1)(2k+1)}{6}+(k+1)^2$ (using (1))

$=(k+1)\left[\frac{k(2k+1)}{6}+(k+1)\right]$

$=(k+1)\left[\frac{2k^2+7k+6}{6}\right]$

$=\frac{(k+1)(k+2)(2k+3)}{6}=\frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$.

Thus $P(k+1)$ is true and the inductive proof is completed.

Hence $P(n)$ is true for all positive integers of $n$.

3. $1^3+2^3+3^3+.......+n^3=\frac{n^2(n+1)^2}{4}$

Answer: Let $P(n):1^3+2^3+3^3+.........+n^3=\frac{n^2(n+1)^2}{4}$

For $n=1$, $P(1):1^3=\frac{1^2(1+1)^2}{4}=\frac{1(2)^2}{4}=\frac{4}{4}=1\Rightarrow 1=1$, is true.

Assume that $P(k)$ is true for some positive integer $k$, we have,

$1^3+2^3+3^3+........+k^2=\frac{k^2(k+1)^2}{4}$ ………..(1)

Now we shall prove that $P(k+1)$ is also true, now we have

$1^3+2^3+3^3+......+k^3+(k+1)^3=\frac{k^2(k+1)^2}{4}+(k+1)^3$ (using (1))

$=(k+1)^2\left[\frac{k^2}{4}+(k+1)\right]$

$=(k+1)^2\left[\frac{k^2+4k+4}{4}\right]$

$=\frac{(k+1)^2(k+2)^2}{4}=\frac{(k+1)^2(k+1+1)^2}{4}$

Thus $P(k+1)$ is also true.

Hence by the principle of mathematical induction $(PMI)$ the statement $P(n)$ is true for all positive integers of $n$.

4. **Prove by the principle of mathematical induction that the sum of the first $n$ odd natural numbers is $n^2$.**

Answer: Let $P(n):1.2+3.4+5.6+.........+(2n-1)=n^2$

For $n=1$, $P(n):1=1^2=1$, which is true

Assume that $P(k)$ is true for some positive integer $k$, we have,

$1+2+3+..................+(2k-1)=k^2$ ………….(1)

We shall now prove that $P(k+1)$ is true, now we have,

$1+2+3+..........+(2k-1)+(2k+1)=k^2+(2k+1)$ (using (1))

$=k^2+2k+1=(k+1)^2$

Thus $P(k+1)$ is also true.

Hence by the principle of mathematical induction (PMI) the statement $P(n)$ is true for all positive integers of $n$.

5. $1.2+3.4+5.6+.....$u[ $n$ terms $=\frac{n(n+1)(4n-1)}{3}$

Answer: Let $P(n):1.2+3.4+5.6+....$ up $n$ terms $=\frac{n(n+1)(4n-1)}{3}$

Here the nth term is $=(2n-1)(2n)$, so the equation becomes.

$P(n):1.2+3.4+5.6+......+(2n-1)(2n)=\frac{n(n+1)(4n-1)}{3}$

For $n=1,\ P(1):1.2=\frac{1(1+1)(4.1-1)}{3}=\frac{1(2)(3)}{3}=2\Rightarrow 2=2$, Which is true.

Assume that $P(k)$ is true for some positive integer $k$, we have,

$1.2+3.4+5.6+........+(2k-1)(2k)=\frac{k(k+1)(4k-1)}{3}$ .........(1)

We shall now prove that $P(k+1)$ is true, now we have,

$1.2+3.4+5.6+.....+(2k-1)(2k)+(2k+1)2(k+1)$

$=\frac{k(k+1)(4k-1)}{3}+(2k+1)2(k+1)$ (using (1))

$=(k+1)\left[\frac{k(4k-1)}{3}+2(2k+1)\right]$

$=(k+1)\left[\frac{\left[4k^2-k+12k+6\right]}{3}\right]=(k+1)\left[\frac{\left(4k^2+11k+6\right)}{3}\right]$

$=\frac{(k+1)(k+2)(4k+3)}{4}=\frac{(k+1)(k+1+1)(4k-1)}{3}$

Thus $P(k+1)$ is true whenever $P(k)$ is true.

Hence by the principle of mathematical induction (PMI) the statement $P(n)$ is true for all positive integers of $n$.

6. $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+............+\frac{1}{(3n-2)(3n+1)}=\frac{n}{(3n+1)}$

Answer: Let $P(n):\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+..........+\frac{1}{(3n-2)(3n+1)}=\frac{n}{(3n+1)}$

For $n=1,\ P(1):\frac{1}{1.4}=\frac{1}{(3.1+1)}=\frac{1}{(4)}\Rightarrow\frac{1}{4}=\frac{1}{4},$ is true.

Assume that $P(k)$ is true of some positive integer $k$, we have,

$\frac{1}{1.4}+\frac{1}{4+7}+\frac{1}{7.10}+......+\frac{1}{(3k-2)(3k+1)}=\frac{k}{3k+1}$

We shall now prove that $P(k+1)$ is true, now we have,

$\frac{1}{1.4}+\frac{1}{4+7}+\frac{1}{7.10}+......+\frac{1}{(3k-2)(3k+1)}=\frac{1}{(3k+1)(3k+4)}=\frac{k}{(3k+1)}+\frac{1}{(3k+1)(3k+4)}$

$=\frac{1}{(3k+1)}\left[k+\frac{1}{3k+4}\right]$

$=\frac{1}{(3k+1)}\left(\frac{3k^2+4k+1}{3k+4}\right)$

$=\frac{1}{(3k+1)}\frac{(k+1)(3k+1)}{3k+4}=\frac{(k+1)}{(3(k+1)+1)}$

Thus $P(k+1)$ is true whenever $P(k)$ is true.

Hence by the principle of mathematical induction (PMI) the statement $P(n)$ is true for all positive integers of $n$.

7. $1.2+2.2^2+3.2^3+............+n.2^n=(n-1)2^{n+1}+2$

Answer: Let $P(n):1.2+2.2^2+3.2^3+......+n.2^n=(n-1)2n^{n+1}+2$

Assume that $P(k)$ is true for some positive integer $k$, we have,

$1.2+2.2^2+3.2^3+......k.2^k+(k+1).2^{k+1}$ ....(1)

We shall now prove that $P(k+1)$ is true, for we have.

$1.2+2.2^2+3.2^3+.........+k.2^k+(k+1).2^{k+1}$

$=(k-1)2^{k+1}+2+(k+1).2^{k+1}$ (using (1))

$=(k-1+k+1)2^{k+1}+2$

$=2k.2^{k+1}=k.2^{k+2}+2$

$=(k+1-1)2^{k+1+1}+2$

Thus $P(k+1)$ is true whenever $P(k)$ is true.

Hence by the principle of mathematical induction (PMI) the statement $P(n)$ is true for all positive integers of $n$.

8. $\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)........\left(1+\frac{2n+1}{n^2}\right)=(n+1)^2$

Answer: Let $P(n):\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right).....\left(1+\frac{2n+1}{n^2}\right)=(n+1)^2$

For $n=1$, $P(1):\left(1+\frac{3}{1}\right)=(1+1)^2=4\Rightarrow 4=4$, Which is true.

Assume that $P(k)$ is true of r some positive integer $k$, we have

$$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right).......\left(1+\frac{2k+1}{k^2}\right)=(k+1)^2 \quad ......(1)$$

Now we shall prove that $P(k+1)$ is also true, now we have,

$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right).......\left(1+\frac{2k+1}{k^2}\right)\left(1+\frac{2k+3}{(k+1)^2}\right)$

$=(k+1)^2\left(1+\frac{2k+3}{(k+1)^2}\right)$ (using (1))

$=k^2+2k+1+2k+3$

$=k^2+4k+4$

$=(k+2)^2=(k+1+1)^2$

Thus $P(k+1)$ is true whenever $P(k)$ is true.

Hence by the principle of mathematical induction (PMI) the statement $P(n)$ is true for all positive integers of $n$.

9. $\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+.....+\frac{1}{(3n-1)(3n+2)}=\frac{n}{6n+4}$

Answer: Let $P(n):\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+...............+\frac{1}{(3n-1)(3n+2)}=\frac{n}{6n+4}$

For $n=1$, $P(1):\frac{1}{2.5}=\frac{1}{6.1+4}=\frac{1}{10}\Rightarrow\frac{1}{10}=\frac{1}{10}$, is true

Assume that $P(k)$ is true for some positive integer $k$, we have,

$$\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+......+\frac{1}{(3k-1)(3k+2)}=\frac{k}{6k+4} \quad ...(1)$$

Now we shall prove that $P(k+1)$ is also true, now we have,

$$\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+.............+\frac{1}{(3k-1)(3k+2)}+\frac{1}{(3k+2)(3k+5)}$$

$$=\frac{k}{6k+4}+\frac{1}{(3k+2)(3k+5)} \quad \text{(using (1))}$$

$$=\frac{k}{2(3k+2)}+\frac{1}{(3k+2)(3k+5)}$$

$$=\frac{1}{3k+2}\left[\frac{k}{2}+\frac{1}{3k+5}\right]$$

$$=\frac{1}{3k+2}\left[\frac{3k^2+5k+2}{2(3k+5)}\right]=\frac{(k+1)(3k+2)}{2(3k+2)(3k+5)}$$

$$=\frac{k+1}{2(3k+5)}=\frac{k+1}{6k+10}=\frac{k+1}{6(k+1)+4}$$

Thus $P(k+1)$ is true whenever $P(k)$ is true.

Hence by the principle of mathematical induction (PMI) the statement $P(n)$ is true for all positive integers of $n$.

10. $1+2+3+.......+n<\frac{(2n+1)^2}{8}$.

Answer: Let $P(n):1+2+3+.....+n<\frac{(2n+1)^2}{8}$

Assume that $P(k)$ is true for some positive integer $k$, we have,

$$1+2+3+......+k<\frac{(2k+1)^2}{8} \quad ............(1)$$

Now we shall prove that $P(k+1)$ is also true, now we have,

$$1+2+3+..........+k+(k+1)<\frac{(2k+1)^2}{8}+(k+1) \quad \text{(using (1))}$$

$$<\frac{1}{8}\left[4k^2+4k+1+8k+8\right]$$

$$<\frac{1}{8}\left[4k^2+12k+9\right]$$

$$<\frac{1}{8}\left[(2k+3)^2\right]$$

$$<\frac{1}{8}\left[((k+1)+1)^2\right]$$

Thus $P(k+1)$ is true whenever $P(k)$ is true.

Hence by the principle of mathematical induction (PMI) the statement $P(n)$ is true for all positive integers of $n$.

11. $10^{2n-1}+1$ **is divisible by 11.**

Answer: Let $P(n):10^{2n1}+1$ is divisible by 11.

$\therefore\ P(1)$ is true.

Assume that $P(k)$ is true for some positive integer $k$, we have, $P(k): 10^{2k-1}+1$ is divisible by 11.

$\Rightarrow 10^{2k-1}+1=11$d.........(1), for some $d \in N$

Now we shall prove that $P(k+1)$ is divisible by 11, we have,

$10^{2(k+1)-1}+1=10^{2k-1+2}+1=10^{2k-1}.10^2+1=10^{2k-1}.100+1$

$=(11d-1)100+1 \qquad (\because \text{from } (1))$

$=1100d-99=11(100d-9)=11\text{m}$, where $m=100d-9 \in N$

Thus $P(k+1)$ is also true.

Hence by PMI, $P(n)$ is divisible by 11 for all $n \in N$

12. $x^{2n}-y^{2n}$ **is divisible by** $(x+y)$

Answer: Let $P(n): x^{2n}-y^{2n}$ is divisible by $(x+y)$

For $n=1$, $P(1): x^{2.1}-y^{2.1}=x^2-y^2=(x+y)(x-y)$ is divisible by $(x+y)$.

$\therefore\ P(1)$ is true.

Assume that $P(k)$ is true for some positive integer $k$, we have,

$P(k): x^{2k}-y^{2k}$, is divisible by $(x+y)$

$P(k): x^{2k}-y^2=(x+y)d$ ......... (1), for some $d \in N$

Now we shall prove that $P(k+1)$ is divisible by $x+y$, we have

$x^{2(k+1)}-y^{2(k+1)}=x^{2k+2}-y^{2k+2}=x^{2k}.\,x^2-y^{2k}.y^2$

$=x^{2k}\cdot x^2-x^2\cdot y^{2k}+x^2\cdot y^{2k}-y^{2k}.y^2 \qquad$ (add & sub. $x^2.y^{2k}$ )

$=x^2\left(x^{2k}-y^{2k}\right)+y^{2k}\left(x^2-y^2\right)$

$=x^2.(x+y)d+y^{2k}(x-y)(x+y) \qquad (\because$ from (1)),

$=(x+y)\left[x^2d+y^{2k}(x-y)\right]$

$=(x+y)m$, where $m=x^2d+y^{2k}(x-y) \in N$

Thus $P(k+1)$ is also true.

Hence by PMI, $P(n)$ is divisible by all $n \in N$