Blue Print (As per PU Board)

Topic	1 mark questions	2 marks questions	3 marks questions	5 marks questions	Total Marks
Complex Numbers & Quadratic Equation	1	1	2	–	9

One mark questions

1. **Write the complex conjugate of** $(1+i)^2$

   Answer: $Z=(1+i)^2=1-1+2i=2i$

   $\vec{Z}=-2i$

2. **Evaluate:** $i^{18}+\left(\frac{1}{i}\right)^{25}$

   Answer: $i^{18}+\left(\frac{1}{i}\right)^{25}=i^{16}\cdot i^2+(-i)^{25}=-1-i^{24}\cdot i=-1-i$

3. **Find the modulus of** $\frac{2-i}{5i}$

   Answer: $Z=\frac{2-i}{5i}\;\therefore |z|=\frac{|2-i|}{|5i|}=\frac{\sqrt{5}}{5}=\frac{1}{\sqrt{3}}$

4. **Find the amplitude of** $1+i$

   Answer: $Z=1+i=r(\cos\theta+i\sin\theta)$

   $$\left.\begin{aligned} r=\sqrt{2},\ \cos\theta&=\frac{1}{\sqrt{2}}\\ \sin\theta&=\frac{1}{\sqrt{2}}\end{aligned}\right\}$$

Two marks questions

5. **If** $x+iy=\frac{a+ib}{a-ib}$ **Prove that** $x^2+y^2=1$

   Answer: $x+iy=\frac{a+ib}{a-ib}$

   $\therefore x-iy=\frac{a-ib}{a+ib}$

   $\therefore (x+iy)(x-iy)=\frac{a+ib}{a-ib}\times\frac{a-ib}{a+ib}$

   $x^2+y^2=1$

6. **If** $1+4\sqrt{3}i=(a+ib)^2$ **prove that** $a^2-b^2=1$ **and** $ab=2\sqrt{3}$

   Answer: $1+4\sqrt{3}i=(a+ib)^2=\left(a^2-b^2\right)+2iab$ equating real and imaginary parts. $a^2-b^2=1,\ ab=2\sqrt{3}$

7. **Find the least +ve integer** m **such that** $\left(\frac{1+i}{1-i}\right)^{2m}=1$

   Answer: $\left(\frac{1+i}{1-i}\right)^{2m}=1\Rightarrow\left[\frac{(1+i)(1+i)}{(1+i)(1+i)}\right]^{2m}=1\Rightarrow\left(\frac{1-1+2i}{2}\right)^{2m}=1$

$\Rightarrow i^{2m} = 1 = i^4$

$\Rightarrow 2m = 4 \Rightarrow m = 2$

8. **If** $x - iy = \sqrt{\frac{a-ib}{c-id}}$ **. Prove that** $\left(x^2+y^2\right)^2 = \frac{a^2+b^2}{c^2+d^2}$

Answer: $x - iy = \sqrt{\frac{a-ib}{c-id}}$

$\therefore (x-iy)^2 = \frac{a-ib}{c-id}$

$\left(x^2-y^2\right) - 2i\,xy = \frac{a-ib}{c-id}$

$\left|\left(x^2-y^2\right) - 2i\;xy\right| = \left|\frac{a-ib}{c-id}\right|$

$\sqrt{\left(x^2-y^2\right)^2 + 4x^2y^2} = \frac{|a-ib|}{|c-id|}$

$\sqrt{x^4+y^4-2x^2y^2+4x^2y^2+4x^2y^2} = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$

$\sqrt{x^4+y^4+2x^2y^2} = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$

$\sqrt{\left(x^2+y^2\right)^2} = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$

Sq. on both sides.

$\left(x^2+y^2\right)^2 = \frac{a^2+b^2}{c^2+d^2}$

Three marks questions

9. **Express complex numbers in Polar form:**

(a) $1-i$ **(b)** $-1+i$ **(c)** $\sqrt{3}+i$ **(d)** $\frac{1-i\sqrt{3}}{2}$ **(e)** $\frac{-1+i\sqrt{3}}{2}$ **(f)** $\frac{1+3i}{1-2i}$

Answer: (a) Let $Z = 1 - i = r(\cos\theta + I\sin\theta)$

$r = \sqrt{1+1} = \sqrt{2}$

$$\left.\begin{aligned}\cos\theta &= \frac{1}{\sqrt{2}}\\ \sin\theta &= -\frac{1}{\sqrt{2}}\end{aligned}\right\}\theta = -\frac{\pi}{4}$$

$1 - i = \sqrt{2}\left[\cos\left(\frac{-\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$

(b) Let $Z = -1 + i = (\cos\theta + I\sin\theta)$

$r = \sqrt{1+1} = \sqrt{2}$

$$\left.\begin{aligned}\cos\theta &= -\frac{1}{\sqrt{2}}\\ \sin\theta &= \frac{1}{\sqrt{2}}\end{aligned}\right\}\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$-1+i=\sqrt{2}\left(\cos\frac{3\pi}{4}+\sin\frac{3\pi}{4}\right)$

(c) Let $Z=\sqrt{3}+i$

$r=\sqrt{3+1}=2$

$\left.\begin{array}{l}\cos\theta=\frac{\sqrt{3}}{2}\\ \sin\theta=\frac{1}{\sqrt{2}}\end{array}\right\}\theta=\frac{\pi}{6}$

$\sqrt{3}+i=2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

(d) Let $Z=\frac{1-i\sqrt{3}}{2}$

$r=\sqrt{\frac{1}{4}+\frac{3}{4}}=1$

$\left.\begin{array}{l}\cos\theta=\frac{1}{2}\\ \sin\theta=-\frac{\sqrt{3}}{2}\end{array}\right\}\theta=-\frac{\pi}{3}$

$\frac{1-i\sqrt{3}}{2}=\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)$

(e) Let $Z=\frac{1-i\sqrt{3}}{2}$

$r=\sqrt{\frac{1}{4}+\frac{3}{4}}=1$

$\left.\begin{array}{l}\cos\theta=-\frac{1}{2}\\ \sin\theta=+\frac{\sqrt{3}}{2}\end{array}\right\}\theta=\pi-\frac{\pi}{3}=2\frac{\pi}{3}$

$\frac{-1+i\sqrt{3}}{2}=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$

(f) Let $Z=\frac{1+3i}{1-2i}\times\frac{1+2i}{1+2i}$

$=\frac{1+2i+3i-6}{1+4}=\frac{-5+5i}{5}$

$=-1+i$

$r=\sqrt{1+1}=\sqrt{2}$

$\left.\begin{array}{l}\cos\theta=-\frac{1}{\sqrt{2}}\\ \sin\theta=\frac{1}{\sqrt{2}}\end{array}\right\}\theta=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$

$Z=\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$

10. **Find the conjugate of:** $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

Answer: $Z = \frac{(3-2i)(2+3i)}{(1+2i)(2-i)} = \frac{6+9i-4i+6}{2-i+4i+2} = \frac{12+5i}{4+3i}$

$= \frac{(12+5i)(4-3i)}{16+9} = \frac{48-36i+20i+15}{25}$

$= \frac{63-16\,i}{25}$

$\bar{Z} = \frac{63+16\,i}{25}$

11. **If** $p+iq = \frac{(a-i)^2}{2a-i}$ **show that** $p^2+q^2 = \frac{(a^2-1)^2}{4a^2+1}$

Answer: $P \to pq = \frac{(\alpha-i)^2}{2\alpha-i}$

$\therefore\ p-iq = \frac{(\alpha+i)^2}{2\alpha+i}$

$(p+iq)(p-iq) = \frac{(\alpha-i)^2}{2\alpha-i}\cdot\frac{(\alpha+i)^2}{2\alpha+i}$

$p^2+q^2 = \frac{(\alpha^2+1)}{4\alpha^2+1}$

12. **Solve: (a)** $x^2+3x+9=0$ **(b)** $3x^2-4x+\frac{20}{3}=0$ **(c)** $27x^2-10x+1=0$ **(d)** $ix^2-x+12i=0$

Answer: (a) $x^2+3x+9=0$

$x = \frac{-3\pm\sqrt{9-36}}{2} = \frac{-3\pm\sqrt{-27}}{2}$

$= \frac{-3\pm i3\sqrt{3}}{2}$

(b) $3x^2-4x+\frac{20}{3}=0$

$9x^2-12x+20=0$

$x = \frac{+12\pm\sqrt{144-720}}{18} = \frac{12\pm\sqrt{-576}}{18} = \frac{12\pm 24i}{18}$

$= \frac{2\pm 4i}{3}$

(c) $27x^2-10x+1=0$

$x = \frac{10\pm\sqrt{100-108}}{2\times 27} = \frac{10\pm 2\sqrt{2}t}{2\times 27}$

$= \frac{5\pm\sqrt{2}i}{27}$

(d) $ix^2-x+12i=0$

$x = \frac{1\pm\sqrt{1+48}}{2i} = \frac{1\pm 7}{2i}$

$x = \frac{8}{2i}\ x = \frac{-6}{2i}$

$x = -4i\ x = 3i$