Blue Print (As per PU Board)

Topic	1 mark questions	2 marks questions	3 marks questions	5 marks questions	Total Marks
Binomial Theorem	–	–	1	1	8

Two mark questions

1. **Simplify $\left(\sqrt{3}+1\right)^6+\left(\sqrt{3}-1\right)^6$**

   Answer: Consider

   $\left(\sqrt{3}+1\right)^6=\left(\sqrt{3}\right)^6+{}^6C_1\left(\sqrt{3}\right)^5+{}^6C_2\left(\sqrt{3}\right)^4+{}^6C_3\left(\sqrt{3}\right)^3+{}^6C_4\left(\sqrt{3}\right)^2+{}^6C_5\left(\sqrt{3}\right)+1$

   $\left(\sqrt{3}+1\right)^6=33+6\left(9\sqrt{3}\right)+15(9)+20\left(3\sqrt{3}\right)+15(3)+6\sqrt{3}+1$

   $\left(\sqrt{3}-1\right)^6=33-54\sqrt{3}+135-60\sqrt{3}+45-6\sqrt{3}+1$

   $\left(\sqrt{3}+1\right)^6+\left(\sqrt{3}-1\right)^6=2\left(3^3\right)+2(135)+2(45)+2$

   $=2[27+135+45+1]$

   $=2[208]$

   $=416$

2. **Using binomial theorem evaluate $(0.99)^6$ correct to four decimal places.**

   Answer: $(0.99)6=(1-0.01)6$

   $=1-6C_1(0.01)+6C_2(0.01)^2-6C_3(0.01)^3+6C_4(0.01)^4-6C_5(0.01)^5+(0.01)^6$

   $=1-0.06+15(0.0001)-20(0.000001)+$ …..(neglecting higher powers of 0.01)

   $=0.9415$

3. **Find the 12th term in the expansion of $\left(\frac{2}{y}-x\right)^{20}$**

   Answer: $T_{r+1}={}^{20}C_r\left(\frac{2}{y}\right)^{20-r}(-x)^r$

   Put $r=11$,

   $T_{11+1}={}^{20}C_1\left(\frac{2}{y}\right)^9(-x)^{11}$

   $T_{12}=-{}^{20}C_9\cdot\frac{2^9\cdot x^{11}}{y^9}$

4. **Find the middle term in the expansion of $\left(2a-\frac{a^3}{6}\right)^{10}$**

   Answer: $T_{r+1}={}^{10}C_r(2a)^{10-r}\left(\frac{-a^3}{6}\right)^r$

   $n=10$ $\therefore$ the middle term is $T_{\frac{n}{2}+1}=T6$

   Putting $r=5$, $T_{5+1}={}^{10}C_5(2a)^{10-5}\left(\frac{-a^3}{6}\right)^5$

$T_6 = {}^{10}C_5 2^5 a^5 \left(-\frac{a^{15}}{6^5}\right)$

$= -\frac{(252)(2^5)a^{20}}{2^5 \times 3^5}$

$= -\frac{252}{243}a^{20}$ is the middle term

5. **Find the term independent of $'x'$ in the expansion of $\left(\frac{x^2}{2} - \frac{1}{3x^3}\right)^{10}$**

Answer: $T_{r+1} = {}^{10}C_r \left(\frac{x^2}{2}\right)^{10-r} \left(-\frac{1}{3x^3}\right)^r$

$= {}^{10}C_r \frac{x^{20-2r}}{2^{10-r}} \frac{(-1)^r}{3^r x^{3r}}$

$T_{r+1} = {}^{10}C_r \frac{(-1)^r}{2^{10-r}3^r} x^{20-5r}$

Equating the power of x to zero

$20 - 5r = 0$

$\rightarrow r = 4$

From equation (1)

$T_{4+1} = {}^{10}C_4 \frac{(-1)4}{2^6 3^4}$ $\quad\quad {}^{10}C_4 = \frac{10\times9\times8\times7}{4\times3\times2\times1} = 210$

$T_5 = \frac{35}{864}$ is the constant term

6. **Prove that the coefficients of x^m and x^n are equal in the expansion of $(1+x)^{m+n}$ where $'m'$ and $'n'$ are positive integers.**

Answer: $T_{r+1} = {}^{(m+n)}C_r (1)^{m+n-r} x^r$

$T_{r+1} = {}^{(m+n)}C_r x^r$

Coefficient of x^m is ${}^{(m+n)}C_m$

Coefficient of x^n is ${}^{m+n}C_n$

But ${}^{(m+n)}C_m = {}^{(m+n)}C_{m+n-m}$

${}^{(m+n)}C_m = {}^{(m+n)}C_n$

Hence proved

Five marks questions

7. **The 3rd, 4th and 5th terms in the expansion of $(x+a)^n$ are respectively 84, 280 and 560. Find the values of x, a and n**

Answer: $T_{r+1} = {}^nC_r X^{n-r} a^r$

Given $T_3 = 84, T_4 = 280, T_5 = 560$

Now $T_3 = 84$

${}^nC_2 x^{n-2} a^2 = 84$

$\frac{n(n-1)(x)^{n-2}a^2}{2x1} = 84$

$n(n-1)x^{n-2}a^2 = 84 \times 2 \qquad \rightarrow (1)$

$T_4 = 280$

${}^nC_3\ x^{n-3}\ a^3 = 280$

$\frac{n(n-1)(n-2)}{3\times2\times1}x^{n-3}a^3 = 280$

$n(n-1)(n-2)x^{n-3}a^3 = 280\times6 \qquad \rightarrow (2)$

$T_5 = 560$

${}^nC_4\ x^{n-4}\ a^4 = 560$

$\frac{n(n-1)(n-2)(n-3)}{4\times3\times2\times1}x^{n-4}\ a^4 = 560$

$n(n-1)(n-2)(n-3)x^{n-4}a^4 = 560\times24 \qquad \rightarrow (3)$

$\frac{(3)}{(2)}$

$\frac{(n-3)a}{x} = \frac{560\times24}{280\times6}$

$(n-3)a = 8x \qquad \rightarrow (4)$

$\frac{(2)}{(1)}$

$\frac{(n-2)a}{x} = \frac{280\times6}{84\times2}$

$(n-2)a = 10x \qquad \rightarrow (5)$

$\frac{(4)}{(5)}$

$\frac{n-3}{n-2} = \frac{4}{5}$

$5(n-3) = 4(n-2)$

$n = 7$

Putting $n = 7$ in (4) we get $a = 2x$

Substituting $n = 7$ and $a = 2x$ in (1), we get

$7\times6\times x^{7-2}(2x)^2 = 84\times2$

$x^7 = \frac{84\times2}{7\times6\times4}$

$X^7 = 1$

$\therefore x = 1$

Since $a = 2x \rightarrow a = 2$

$\therefore x = 1,\ a = 2,\ n = 7$

8. **Using binomial theorem prove that $3^{2n} - 8n - 9$ is divisible by 8 where 'n' is a positive integer.**

Answer: Consider $(1+8)^n = 1 + {}^nC_1 8 + {}^nC_2(8)^2 + {}^nC_3\ 8^3 + \ldots\ldots. + 8^n$

$9^n = 1 + 8n + {}^nC_2(8)^2 + {}^nC_3\ 8^3 + \ldots\ldots. + 8^n$

$9^n - 8n - 9 = -8 + {}^nC_2(8)^2 + {}^nC_3\ 8^3 + \ldots\ldots. + 8^n$

$9^n - 8n - 9 = 8\left[-1 + {}^nC_2(8) + {}^nC_3 8^2 + ... + 8^{n-2}\right]$

$9^n - 8n - 9 = 8\ K$

Where $K = -1 + 8(nC_2) + 8^2 \cdot nC_3 + ... + 8^{n-2}$ is an integer.

$\therefore\ 9^n - 8n - 9$ is divisible by 8 i.e. $3^{2n} - 8n - 9$ is divisible by 8

9. **The coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ration $1:7;\ 42$ find $'n'$.**

Answer: $T_{r+1} = {}^nC_r\ x^r$

Let coefficients of T_{r-1}, T_r, T_{r+1} be in the ratio 1 : 7 : 42

$\dfrac{\text{coeff. of } T_{r+1}}{\text{coeff. of } T_r} = \dfrac{1}{7}$ and $\dfrac{\text{coeff. of } T_r}{\text{coeff. of } T_{r+1}} = \dfrac{7}{42}$

$\dfrac{{}^nC_{r-1}}{{}^nC_r} = \dfrac{1}{7}$ and $\dfrac{{}^nC_r}{{}^nC_{r+1}} = \dfrac{1}{6}$

$\dfrac{r}{n-r+1} = \dfrac{1}{7}$ and $\dfrac{r+1}{n-r} = \dfrac{1}{6}$

$7r = n - r + 1$ and $\left[\therefore \dfrac{{}^nC_{k-1}}{{}^nC_k} = \dfrac{k}{n-k+1}\right]$

$6(r+1) = n - r$

$8r = n + 1$ and $7r = n - 6$

Solving these equations we get

$r = 7,\ n = 55$

10. **Find the coefficient of x^{10} in the expansion of $(1+x+x^2)(1-x)^{15}$**

Answer: $(1+x+x^2)(1-x)^{15} = (1+x+x^2)(1-x)(1-x)^{14}$

$= (1-x^3)(1-x)^{14}$

$= (1-x^3)(1 - {}^{14}C_1 x + {}^{14}C_2 x^2 .... + x^{14})$

$\therefore$ The coefficient of $x^{10} = {}^{14}C_{10} - {}^{14}C_7$

$= {}^{14}C_4 - {}^{14}C_7$

$= 1001 - 3432$

$= -2431$

11. **Given that the coefficients of $(2m-1)^{\text{th}}$ and $(m+2)^{\text{th}}$ terms in the expansion of $(1+x)^{43}$ are equal, find $'m'$**

Answer: $T_{r+1} = {}^{43}C_r x^r$

Putting $r = 2m$,

$T_{2m+1} = {}^{43}C_{2m}\ x^{2m}$

For $r = m+1$

$T_{m+2} = {}^{43}C_{m+1}\ x^{m+1}$

Given coefficient of T_{2m+1} = coefficient of T_{m+2}

${}^{43}C_{2m} = {}^{43}C_{m+1}$

$2m = m+1$ or $2m + m + 1 = 43$

$m = 1$ or $m = 14$

12. **Find the term independent of x in the expansion of $\left(1-2x+x^3\right)\left(x-\frac{1}{x}\right)^{15}$**

Answer: Constant term

$=[-2\times\text{coefficient of } x^{-1} \text{ in}\left(x-\frac{1}{x}\right)^{15}+[1\times\text{coefficient of } x^{-3}\text{in}\left(x-\frac{1}{x}\right)^{15} \qquad \rightarrow(1)$

General term of $\left(x-\frac{1}{x}\right)^{15}$ is $T_{r+1}={}^{15}C_r\, x^{15-r}\left(\frac{-1}{x}\right)^r$

$T_{r+1}={}^{15}C_r\left(-1\right)^r\ x^{15-2r}$

If $15-2r=-1 \qquad \rightarrow r=8$

If $15-2r=-3 \qquad \rightarrow r=9$

$\therefore$ coefficient of x^{-1} is ${}^{15}C_8$

And coefficient of x^{-3} is $-{}^{15}C_9$

$\therefore$ from (1)

Constant term $=\left[-2\times 15C_8\right]+\left[1\times\left(-15\ C_9\right)\right]$

$=\left(-2\times 15C_7x\right)+\left(-15\right)C_6$

$=-12870-5005=-17875$