Blue Print (As per PU Board)

Topic	1 mark questions	2 marks questions	3 marks questions	5 marks questions	Total Marks
Gravitation	–	–	1	1	8

One mark questions

1. **State Kepler's law of orbits.**
   Answer: All planets move in elliptical orbits with the sun situated at one of the foci of the ellipse.
2. **State Kepler's law of periods.**
   Answer: The square of the time period of revolution of a planet is proportional to the cube of the semi – major axis of the ellipse traced out by the planet.
3. **Which physical quantity is conserved in the case of law of areas?**
   Answer: "Angular momentum" is conserved in the case of law of areas.
4. **State universal law of gravitation.**
   Answer: Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
5. **What is escape speed?**
   Answer: Escape speed is the minimum speed required for an object to escape from the earth (i.e.to reach infinity (or) zero gravitational potential energy.)

Two marks questions

6. **Define gravitational potential energy of a body. Give an expression for it.**
   Answer: Potential energy of a body arising out of the force of gravity is called the gravitational potential energy. The expression for gravitational potential energy of a body due to earth is
   $V = W(r) = -G\ M_{Em} / r$
   Where $\mathrm{M_E}$ is mass of the earth,
   m is mass of the body,
   r is the distance of the body from the centre of the earth.
7. **Derive the relation between gravitational constant and acceleration due to gravity.**
   Answer: Let m be the mass of the body situated on the surface of the earth of radius R_E and mass M_E
   According to newton's law of gravitation the force between the body close to the surface of the earth and the earth is;
   $F = GmM_E / R_E^2$ …..(1)
   Where G is gravitational constant.
   But according to Newton's 2nd law of motion, the gravitational force exerted on the body by the earth;
   $F = mg$ ….(2)
   Where g is acceleration due to gravity.
   $\therefore$ Form equations (1) & (2)
   $mg = GmM_E / R_E^2$ (or) $g = GM_E / R_E^2$
8. **Explain the state of weightlessness of a body.**
   Answer: Weight of a body is the force with which the earth attracts it. If there is no opposite force (support) exerted on the body it would fall down. During this free fall, the object appears to lose its weight and becomes weightless, since there is no upward force on the body. This state of a body is called weightlessness
9. **An object weighs more on the surface than at the centre of the earth. Why?**
   Answer: The weight of a body at a place on the surface of the earth is given by $W = mg$. since the value of g is maximum on the surface, the object weighs more on the surface. At the centre of the earth, the value of g is zero. Therefore the weight of the body at the centre of the earth is zero.

Five marks questions

10. **Obtain the expression for escape speed.**

Answer: AThe minimum speed required to project an object vertically upwards from the surface of the earth, so that it escapes from the gravitational influence of the earth and never returns to the earth, is called escape speed.

Consider an object of mass m at a distance of r from the centre of the earth.

The gravitational force acting on the mass m is given by

$F = GMm/r^2$ ….(1)

This force acts towards the centre of the earth.

Let dr be the small distance covered by the object away from the centre of the earth.

Therefore the work done on the object against the gravitational force of attraction of the earth is

$dW = \vec{F}\cdot\vec{dr} = F\ dr\ \cos 180° = -F\ dr$

From equation (1) we get, $dW = -\left[GMm/r^2\right]dr$ ……..(2)

Therefore total work done to displace the object from the surface of the earth [i.e., $r = R$] to $[r = \infty]$ is calculated by integrating equation (2) between the limits R and ∞.

Therefore $\int_R^\infty dW = \int_R^\infty -\left[GMm/r^2\right]dr$

$$W = -GMm\left[\frac{r^{-1}}{-1}\right]_R^\infty = GMm\left[\frac{1}{r}\right]_R^\infty = GMm\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

Or $W = -GMm/R$ [since $\frac{1}{\infty} = 0$] this work done is equal to the potential energy (V) of the object of mass m. That is $V = -GMm/R$. Let v_e is the escape speed of the object of mass m then its kinetic energy is K.E = $\frac{1}{2}mv_e^2$. If kinetic energy of the object = magnitude of potential energy of the object then, $\frac{1}{2}Mv_e^2 = GMm/R$ or $v_e^2 = \frac{2GM}{R}$

$$v_e = \sqrt{\frac{2GM}{R}}$$

But $GM/R^2 = g$

Or $GM = g\ R^2$

Therefore $v_e = \sqrt{2gR}$

11. **Derive the expression for orbital speed of a satellite/ period of a satellite around the earth.**

Answer: The speed with which a satellite moves in its orbit around the earth is called orbital speed.

The time taken by a satellite to complete one revolution in its orbit around the earth is called period of a satellite.

Let a satellite of mass m revolves around the earth in an orbit at a height of h from the surface of the earth. If R is the radius of the earth, then the radius of the orbit of the satellite is $R + h$. Let v_0 be the orbital speed of the satellite.

The gravitational force of attraction between the earth and the satellite provides the necessary centripetal force to the satellite to move in a circular orbit around the earth.

i.e., Gravitational force – centripetal force

$$\frac{GMm}{(R+h)^2} = \frac{mv_0^2}{(R+h)}$$

That is $$v_0^2 = \frac{GM}{(R+h)}$$

Therefore $v_0 = \sqrt{\frac{GM}{(R+h)}}$

But $\frac{GM}{R^2} = g$ or $GM = gR^2$

Therefore $v_0 = \sqrt{\frac{gR^2}{(R+h)}}$

If the satellite is very close to the earth. i.e., h<<
Then $(R+h) = R$

Therefore $v_0 = \sqrt{gR}$

Period of a Satellite $T = \frac{\text{circum ference of the orbit}}{\text{orital speed}}$

i.e., $T = \frac{2\pi(R+h)}{v_0}$

we know that $v_0 = \sqrt{\frac{gR^2}{(R+h)}}$

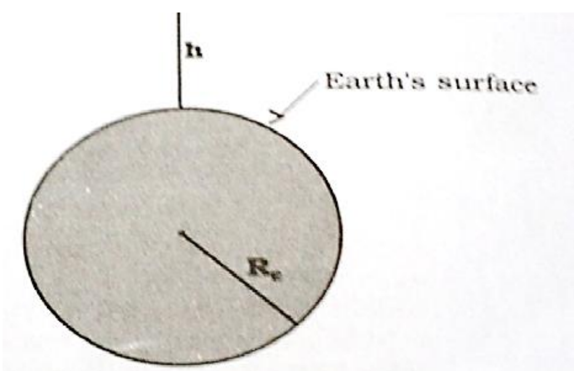
Therefore $T = \frac{2\pi(R+h)}{\sqrt{\frac{gR^2}{(R+h)}}}$

i.e., $T = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$

If the satellites is very close to the earth, h<<
Then $(R+h) = R$

Therefore $T = 2\pi\sqrt{\frac{R}{g}}$

12. **Derive the expression for acceleration due to gravity at a point above the surface of the earth.**
Answer: Consider a point mass m at a height h above the surface of the earth as shown in figure.



Its distance from the centre of the earth is $(R_E - d)$. The earth can be thought of as being composed of a smaller sphere of radius $(R_E - d)$ and a spherical sheel of thickness d. The force on m due to the outer shell of thickness d is zero. The point mass m is outside the smaller sphere of radius $(R_E - d)$. The force due to this smaller sphere is just as if the entire mass of the smallersphere is concentrated at the centre. If M_s is the mass of the smaller sphere, then

$M_s / M_E = (R_E - d)^3 / R_E^3$ …(1)

(Since mass of a sphere is proportional to the cube of its radius).

Therefore the force on the point mass is $F(d) = GM_s m / (R_E - d)^2$ …(2)

Substituting for M_s we get

$F(d) = GM_E m (R_E - d) / R_E^3$ …(3)

Therefore the acceleration due to gravity at a depth d is,

$g(d) = F(d) / m$

That is $g(d) = \left[GM_E / R_E^3\right](R_E - d)$

$= g(R_E - d) / R_E$

$g(d) = g(1 - d / R_E)$ …(4)

Therefore as we go down below earth's surface, the acceleration due to gravity decreases by a factor $(1 - d / R_E)$.