Blue Print (As per PU Board)

Topic	1 mark questions	2 marks questions	3 marks questions	4 marks questions	5 marks questions	Total Marks
Limits & Derivatives	1	1	1	1	1	15

One mark questions

1. **Evaluate** $\lim_{x\to 0}\frac{x^2-5x+9}{x+3x+6}$ **.**

   Answer: $\lim_{x\to 0}\frac{x^2-5x+9}{x+3x+6}$

   $\frac{0+9}{0+6}=\frac{3}{2}$

2. **Differentiate the following w.r.t. 'x'** $x^2\cos x$

   Answer: $Y=x^2\cos x$

   $\frac{dy}{dx}=x^2\frac{d}{dx}(\cos x)+\cos x\frac{d}{dx}\left(x^2\right)$

   $=-x^2\sin x+2x\cos x$

3. **Differentiate the following w.r.t. 'x'** $(2x+1)(3x-5)$

   Answer: $Y=(2x+1)(3x-5)$

   $\frac{dy}{dx}=(2x+1)\frac{d}{dx}(3x+5)+(3x-5)\frac{d}{dx}(2x+1)$

   $=(2x+1)(3)+(3x-5)(2)$

   $=6x+3+6x-10$

   $=12x-7$

Two marks questions

4. **Evaluate** $\lim_{x\to 2}\frac{x^2-x-2}{x^2-5x+6}$

   Answer: $\lim_{x\to 2}\frac{(x-2)(x+1)}{(x-2)(x-3)}$

   $=\frac{(2+1)}{(2-3)}=\frac{3}{-1}$

   $=-3$

5. **If** $f(x)=x\tan x$ **then find** $f^1\left(\frac{\pi}{4}\right)$

   Answer: $f(x)=x\tan x$

   $f^1(x)=x\frac{d}{dx}(\tan x)+\tan x\frac{d}{dx}(x)$

   $=x\sec 2x+\tan x$

   $f^1\left(\frac{\pi}{4}\right)=\frac{\pi}{4}\left(\sqrt{2}\right)^2+1=\frac{\pi}{2}+1$

6. **Differentiate the following w.r.t 'x'** $\frac{1-x}{1+x}$

Answer: $y=\frac{1-x}{1+x}$

Diff w.r.t. $'x'$

$$\frac{dy}{dx}=\frac{(1+x)\frac{d}{dx}(1-x)-(1-x)\frac{d}{dx}(1+x)}{(1+x)^2}$$

$$=\frac{(1+x)(-1)-(1-x)(1)}{(1+x)^2}$$

$$=\frac{-1-x-1+x}{(1+x)^2}$$

$$\therefore \frac{dy}{dx}=\frac{-2}{(1+x)^2}$$

Three marks questions

7. **Evaluate** $\lim_{\theta\to\frac{x}{2}}\frac{\cos 3\theta+3\cos\theta}{\left(\frac{\pi}{2}-\theta\right)^3}$

Answer: $\lim_{\theta\to\frac{x}{2}}\frac{\cos 3\theta+3\cos\theta}{\left(\frac{\pi}{2}-\theta\right)^3}$

$$\lim_{\theta\to\frac{x}{2}}\frac{4\cos 3\theta-3\cos\theta+3\cos\theta}{\left(\frac{\pi}{2}-\theta\right)^3}$$

$$\lim_{\theta\to\frac{x}{2}}\frac{4\cos^3\theta}{\left(\frac{\pi}{2}-\theta\right)^3}$$

put $\frac{\pi}{2}-\theta=t$ As $\theta\to\frac{\pi}{2}=t\to 0$

$$\lim_{\theta\to\frac{x}{2}}\frac{4\left[\cos\left(\frac{\pi}{2}-t\right)\right]^3}{t^3}$$

$$\lim_{\theta\to 0}\frac{4\sin^3 t}{t^3}$$

$=4\times 1$

$= 4$

8. **Differentiate the following w.r.t 'x' from first principles** $\frac{1}{x}$.

Answer: $f(x)=\frac{1}{x}$

$$f^1(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{\frac{1}{x+\Delta x}-\frac{1}{x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x(x+\Delta x)}{x(x+\Delta x)\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{x(x+\Delta x)\Delta x}$$

$$= -\frac{1}{x(x+0)}$$

$$= -\frac{1}{x^2}$$

9. **Differentiate the following w.r.t. 'x'** $\frac{2+3\cos x}{3-2\sin x}$

Answer: $y = \frac{2+3\cos x}{3-2\sin x}$

Diff w.r.t. 'x'

$$\frac{dy}{dx} = \frac{(3-2\sin x)\frac{d}{dx}(2+3\cos x)-(2+3\cos x)\frac{d}{dx}(3-2\sin x)}{(3-2\sin x)^2}$$

$$= \frac{(3-2\sin x)(-3\sin x)-(2+3\cos x)(-2s\cos x)}{(3-2\sin x)^2}$$

$$= \frac{-9\sin x+6\sin^2 x+4\cos x+6\cos^2 x}{(3-2\sin x)^2}$$

$$= \frac{dy}{dx} = \frac{6+4\cos x-9\sin x}{(3-2\sin x)^2}$$

Five marks questions

10. **Evaluate** $\lim_{x\to 0} \frac{|x|}{x}$ **if its exists.**

Answer: $|x| = \begin{cases} x & if \quad x>0 \\ 0 & if \quad x=0 \\ -x & if \quad x<0 \end{cases}$

$$\text{RHL} = \underset{x\to 0^+}{Lt} \frac{|x|}{x} = \underset{x\to 0^+}{Lt} \frac{\not{x}}{\not{x}} = 1$$

$$\text{LHL} = \underset{x\to 0^-}{Lt} \frac{|x|}{x} = \underset{x\to 0^-}{Lt} = \frac{x}{x} = 1$$

$$\text{LHL} \neq \text{RHL}$$

$$\therefore \underset{x\to 0}{Lt} \frac{|x|}{x}$$

11. **Differentiate** $\frac{(3x^2+2)\sin x}{(1+x\cos x)}$ **w.r.t 'x'**

Answer: $f(x) = \frac{(3x^2+2)\sin x}{1+x\cos x}$

$$f1(x) = \frac{(3x^2+2)\sin x\cdot\frac{d}{dx}(1+x\cos x)-(1+x\cos x)\frac{d}{dx}\left((3x^2+2)\sin x\right)}{(1+x\cos x)^2}$$

$$=\frac{\left(3x^2+2\right)\sin x\left[-x\sin x+\cos x\right]-\left(1+x\cos x\right)\left(\left(3x^2+2\right)\cos x+6x\sin x\right)}{\left(1+x\cos x\right)^2}$$

12. **Differentiate $\sqrt{\cos x}$ w.r.t. 'x' from first principles.**

Answer: $y=\sqrt{\cos x}$

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{\cos(x+h)}-\sqrt{\cos x}}{h}$$

$$=\lim_{h\to 0}\frac{\left(\sqrt{\cos(x+h)}-\sqrt{\cos x}\right)\left[\sqrt{\cos(x+h)}+\sqrt{\cos x}\right]}{h\left[\sqrt{\cos(x+h)}+\sqrt{\cos x}\right]}$$

$$=\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h\left[\sqrt{\cos(x+h)}+\sqrt{\cos x}\right]}$$

$$=\lim_{h\to 0}\frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h\left[\sqrt{\cos(x+h)}+\sqrt{\cos x}\right]}$$

$$=\lim_{h\to 0}\frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{\left[\sqrt{\cos(x+h)}+\sqrt{\cos x}\right]\left(\frac{h}{2}\right)\times 2}$$

$$=\frac{\sin x\times 1}{\left[\sqrt{\cos x}+\sqrt{\cos x}\right]}$$

$$=\frac{\sin x}{2\sqrt{\cos x}}$$