Blue Print (As per PU Board)

Topic	1 mark questions	2 marks questions	3 marks questions	5 marks questions	Total Marks
Probability	1	1	1	1	11

One mark questions

1. **Two dice are thrown. Find the probability that the sum of the numbers coming up on them is 8, given that the number 3 always appears on the first die.**
   Answer: Given that number 3 always appears on the first die the outcomes become
   $\{(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\}$
   Among these the sum 8 is obtained in $\{(3,5)\}$ only $\therefore$ probability $=\frac{1}{6}$.
2. **In a class 50% failed in Physics, 30% failed in Mathematics and 20% failed in both. If a student is selected at random, find the probability that he will fail in Physics if he has failed in Mathematics.**
   Answer: Assume that number of students in the class $=100$
   Clearly 50 students failed in Physics, 30 failed in Mathematics and 20 failed in both.
   $\therefore$ If a student has failed in Maths, then he is one among the 30 who have done so and if the student is also to fail in Physics he should be one among the 20 who have failed in both.
   $\therefore$ Required probability $=\frac{20}{30}=\frac{2}{3}$.
3. **If $P(A)=\frac{1}{3}$, $P(B)=\frac{1}{12}$ and $P(A\cap B)=\frac{1}{36}$, are the events A and B independent?**
   Answer: For independent events, $P(A\cap B)=P(A)P(B)$
   $P(A\cap B)=\frac{1}{36}$, $P(A)P(B)=\frac{1}{3}\times\frac{1}{12}=\frac{1}{36}$
   $\therefore A$ and B are independent events.

Two marks questions

4. **If $P(A)=0\cdot6$, $P(B)=p$ and $P(A\cup B)=0\cdot8$, and the events A and B are independent, find $P(B)$.**
   Answer: For independent events, $P(A)P(B)=P(A\cap B)=0.6p$ (1 mark)
   $P(A\cup B)=P(A)+P(B)-P(A\cap B)=P(A)+P(B)-P(A)P(B)$
   $0\cdot8=0\cdot6+p-0\cdot6p\Rightarrow p(1-0\cdot6)=0\cdot8-0\cdot6$
   $p(0\cdot4)=0\cdot2 \quad \therefore p=\frac{0\cdot2}{0\cdot4}=\frac{1}{2}$ (1 mark)
5. **If $P(A)=0\cdot6$, $P(B)=0\cdot9$ and $P(B/A)=0\cdot9$ find $P(A\cup B)$**
   Answer: $P(B/A)=\frac{P(A\cap B)}{P(A)}\Rightarrow 0\cdot9=\frac{P(A\cap B)}{0\cdot6}$
   $\therefore P(A\cap B)=0\cdot54$ (1 mark)
   $P(A\cup B)=P(A)+P(B)-P(A\cap B)$
   $=0\cdot6+0\cdot9-0\cdot54=1\cdot5-0\cdot54=0\cdot96$ (1 mark)
6. **Find k, if the following distribution represents a probability distribution.**

x	0	1	2	3	4
$P(x)$	$2k$	k	$3k$	$4k$	$2k$

Answer: W.k.t. for a probability distribution $\sum P(x)=1$ (1 mark)

$\therefore 2k+k+3k+4k+2k=1$

$\therefore 12k=1$

$\therefore k=\frac{1}{12}$ (1 mark)

Three marks questions

7. **A random variable x has the following probability distribution**

x	-2	-1	0	1	2	3
$P(x)$	$0\cdot1$	k	$0\cdot2$	$2k$	$0\cdot3$	k

find the value of (i) k **(ii)** $P(x\le1)$ **(iii)** $P(x\ge0)$

Answer: (i) $0\cdot1+k+0\cdot2+2k+0\cdot3+k=1$

$4k=1-0\cdot6$

$\therefore k=\frac{0\cdot4}{4}=0\cdot1$ (1 mark)

(ii) $P(x\le1)$

$P(x\le1)=P(x=-2)+P(x=-1)+P(x=0)+P(x=1)$

$=0\cdot1+k+0\cdot2+2k$

$=0\cdot3+3k=0\cdot3+3(0\cdot1)=0\cdot6$ (1 mark)

(iii) $P(x\ge0)=P(x=0)+P(x=1)+P(x=2)+P(x=3)$

$=0\cdot2+2k+0\cdot3+k$

$=0\cdot5+3k=0\cdot5+3(0\cdot1)=0\cdot8$ (1 mark)

8. **If** $P(A)=0\cdot8$, $P(B)=0\cdot5$ **and** $P(B/A)=0\cdot4$, **find**

(i) $P(A\cap B)$ **(ii)** $P(A/B)$ **(iii)** $P(A\cup B)$

Answer: (i) $P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}\Rightarrow 0\cdot4=\frac{P(A\cap B)}{0\cdot8}$

$\therefore P(A\cap B)=0\cdot32$ (1 mark)

(ii) $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{0\cdot32}{0\cdot5}=0\cdot064$ (1 mark)

(iii) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=0\cdot8+0\cdot5-0\cdot32$

$=1\cdot3-0\cdot32$

$=0\cdot98$ (1 mark)

9. **A box contains** 30 **apples of which** 10 **are known to be bad. If two apples are randomly picked from the box, what is the probability that both are bad?**

Answer: Sample space $=30$ apples

1 apple is picked from 30 apples out of which 10 are bad

$\therefore$ Probability (apple being bad) $=\frac{10}{30}=\frac{1}{3}$ (1 mark)

$\therefore$ The number of apples in the box reduces to 29. Bad ones will be 9 for the 2^{nd} draw.

$\therefore$ P (2^{nd} apple picked being bad) $=\frac{9}{29}$ (1 mark)

$\therefore$ P (both apples being bad) $=\frac{1}{3}\times\frac{9}{29}=\frac{3}{29}$ (1 mark)

Five marks questions

10. **Find the probability distribution of the number of successes in 2 tosses of a dice, where a success is defined as a number greater than 4. Also find the mean and variance of the distribution.**

Answer: Let ' E 'be the event of getting a success i.e, of getting a number greater than 4 in the toss of a die

$E=\{5,6\}\cdot n(E)=2$

$\therefore P(E)=\frac{2}{6}=\frac{1}{3},\ P(\overline{E})=1-P(E)=1-\frac{1}{3}=\frac{2}{3}$ (1 mark)

Let 'x' denote the random variable 'number greater than 4'

$\therefore$ The possible values of x are 0,1,2.

$P(x=0)=P(\overline{E}_1\overline{E}_2)=p(\overline{E}_1)P(\overline{E}_2)=\frac{2}{3}\times\frac{2}{3}=\frac{4}{9}$

$P(x=1)=P(E_1\overline{E}_2 \text{ or} \overline{E}_1E_2)=P(E_1)P(\overline{E}_2)+P(\overline{E}_1)P(E_2)$

$=\frac{1}{3}\times\frac{2}{3}+\frac{2}{3}\times\frac{1}{3}=\frac{4}{9}$

$P(x=2)=P(E_1E_2)=P(E_1)P(E_2)=\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$ (1 mark)

Probability distribution of x is

x	0	1	2
$P(x)$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(1 mark)

Calculation of mean and variance

x	P	Px	Px^2
0	$\frac{4}{9}$	0	0
1	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{4}{9}$
2	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{4}{9}$
	$\sum P=1$	$\sum Px=\frac{2}{3}$	$\sum Px^2=\frac{8}{9}$

(1 mark)

Mean $(\mu)=\frac{2}{3}$, variance $=\sum Px^2-\mu^2=\frac{8}{9}-\left(\frac{2}{3}\right)^2=\frac{4}{9}$ (1 mark)

11. **In a bolt factory, machines A, B and C manufacture 60% , 25 % and 15% respectively. Of the total of their output 1% , 2% and 1% are defective bolts. A bolt is drawn at random from the total production and found to be defective. What is the probability that the defective bolt was manufactured by A ?**

Answer: Let E_1, E_2, E_3 be the events of drawing a bolt produced by macnines A, B and C respectively E_1, E_2, E_3 are mutually exclusive and exhaustive events. Let D be the event of drawing a defective bolt.

$\therefore P(E_1)=\frac{60}{100}, P(E_2)=\frac{25}{100}, P(E_3)=\frac{15}{100}$ (1 mark)

$P(D/E_1)=\frac{1}{100}, P(D/E_2)=\frac{2}{100}, P(D/E_3)=\frac{1}{100}$ (1 mark)

P(defective bolt is produced by machine A)

$= P(E_1/D)$

$= \dfrac{P(E_1)P(D/E_1)}{P(E_1)P(D/E_1)+P(E_2)P(D/E_2)+P(E_3)P(D/E_3)}$ (using baye's theorem) (1 mark)

$= \dfrac{\frac{60}{100}\times\frac{1}{100}}{\left(\frac{60}{100}\times\frac{1}{100}\right)+\left(\frac{25}{100}\times\frac{2}{100}\right)+\left(\frac{15}{100}\times\frac{1}{100}\right)}$ (1 mark)

$= \dfrac{60}{(60+50+15)} = \dfrac{60}{125} = \dfrac{12}{25}$ (1 mark)

12. **Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed A reports that a head appears. Find the probability that actually there was head.**

Answer: E_1 = event that coin comes up with a head

E_2 =event that coin comes up with a tail

E_1 and E_2 are mutually exclusive and exhaustive events.

$P(E_1) = P(E_2) = \frac{1}{2}$ (1 mark)

Let E =event that A reports that a head appears

$\therefore P(E/E_1) = P$ (head comes up and A speaks truth)= $\frac{4}{5}$ (1 mark)

$P(E/E_2) =$ P (tail comes up and A tells lie) $= 1-\frac{4}{5} = \frac{1}{5}$ (1 mark)

Required probability $= P(E_1/E)$

From Baye's theorem,

$P(E_1/E) = \dfrac{P(E_1)P(E/E_1)}{P(E_1)P(E/E_1)+P(E_2)P(E/E_2)}$ (1 mark)

$= \dfrac{\frac{1}{2}\times\frac{4}{5}}{\left(\frac{1}{2}\times\frac{4}{5}+\frac{1}{2}\times\frac{1}{5}\right)} = \dfrac{\frac{2}{5}}{\frac{2}{5}+\frac{1}{10}} = \dfrac{\frac{2}{5}}{\frac{5}{10}} = \dfrac{4}{5}$ (1 mark)