Blue Print (As per PU Board)

Topic	1 mark questions	2 marks questions	3 marks questions	5 marks questions	Total Marks
Matrices	1	–	1	1	9

One mark questions

1. **Define the following matrices : Square matrix**

   Answer: A matrix having number of rows equal to numbers of columns is called a square matrix

   (1 mark)

2. **Write the order and number of elements of the following**

   $A = [0\ 1\ 2\ 3]$

   Answer: Order of A is 1×4 number of elements = 4 (1 mark)

3. **If a matrix has 24 elements, what are the possible orders it can have? What if it has 13 elements.**

   Answer: $24 = 1 \times 24$ and 24×1

   $= 2 \times 12$ and 12×2 $= 3 \times 8$ and 8×3

   $= 4 \times 6$ and 6×4

   $\therefore$ There are 8 matrices having 24 elements of the orders $24 \times 1, 1 \times 24, 2 \times 12, 12 \times 2, 3 \times 8, 8 \times 3, 4 \times 6$ and 6×4 (1 mark)

   These are 2 matrices having 13 elements with orders $13 \times 1, 1 \times 13$.

Two marks questions

4. Find the x, y and z from the following

(a) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} x+y+x \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Answer: (a) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

$\therefore\ y = 4,\ x = 1,\ z = 3$ (1 mark)

(b) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

$\therefore x + y + z = 9,\ x + z = 5,\ y + z = 7$

$y + z = 7,\ x + 7 = 9 \Rightarrow x = 2$

$2 + z = 5 \Rightarrow z = 3,\ y + z = 7 \Rightarrow y + 3 = 7 \Rightarrow y = 4$

$x = 2,\ y = 4$ and $z = 3$ (1 mark)

5. **Verify whether (a)** $A=\begin{bmatrix}2 & 5\\5 & 4\end{bmatrix}$ **is symmetric matrix and (b)** $B=\begin{bmatrix}0 & 4\\-4 & 0\end{bmatrix}$ **is skew symmetric**

Answer: (a) $A=\begin{bmatrix}2 & 5\\5 & 4\end{bmatrix}$ $\therefore A'=\begin{bmatrix}2 & 5\\5 & 4\end{bmatrix}\Rightarrow A'=A\therefore A$ is symmetric matrix (1 mark)

(b) $B=\begin{bmatrix}0 & 4\\-4 & 0\end{bmatrix}$, $B'=\begin{bmatrix}0 & -4\\4 & 0\end{bmatrix}\Rightarrow -B'=\begin{bmatrix}0 & 4\\-4 & 0\end{bmatrix}$

$\therefore$ $B=-B'$ or $B'=-B\therefore B$ is skew symmetric (1 mark)

6. **If** $A=\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}$ **and** $B=\begin{bmatrix}4 & 7\\6 & 5\end{bmatrix}$ **show that** $AB\neq BA$

Answer: $AB=\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}\begin{bmatrix}4 & 7\\6 & 5\end{bmatrix}=\begin{bmatrix}8+6 & 14+5\\12+24 & 21+20\end{bmatrix}=\begin{bmatrix}14 & 19\\36 & 41\end{bmatrix}$ …(1) (1 mark)

$BA=\begin{bmatrix}4 & 7\\6 & 5\end{bmatrix}\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}=\begin{bmatrix}8+21 & 4+28\\12+15 & 6+20\end{bmatrix}=\begin{bmatrix}29 & 32\\27 & 26\end{bmatrix}$ …(2)

From (1) & (2) $AB\neq BA$ (1 mark)

Three marks questions

7. **Express** $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ **as sum of symmetric and skew symmetric matrixes**

Answer: Let $A=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ $\therefore A'=\begin{bmatrix}1 & 3\\2 & 4\end{bmatrix}$ (1 mark)

$A+A'=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}+\begin{bmatrix}1 & 3\\2 & 4\end{bmatrix}=\begin{bmatrix}2 & 5\\5 & 8\end{bmatrix}$(Symmetric)

$A-A'=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}-\begin{bmatrix}1 & 3\\2 & 4\end{bmatrix}=\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$(Skew symmetric) (1 mark)

$A=\frac{1}{2}(A+A')+\frac{1}{2}(A-A')$

$A=\begin{bmatrix}1 & 5/2\\5/2 & 4\end{bmatrix}+\begin{bmatrix}0 & -1/2\\1/2 & 0\end{bmatrix}=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ (1 mark)

8. **If** $A=\begin{bmatrix}3 & -5\\-4 & 2\end{bmatrix}$ **then find** $A^2-5A-14I$

Answer: $A^2=\begin{bmatrix}3 & -5\\-4 & 2\end{bmatrix}\begin{bmatrix}3 & -5\\-4 & 2\end{bmatrix}=\begin{bmatrix}29 & -25\\-20 & 24\end{bmatrix}$ (1 mark)

$5A=\begin{bmatrix}15 & -25\\-20 & 10\end{bmatrix}$ and $14I=\begin{bmatrix}14 & 0\\0 & 14\end{bmatrix}$ (1 mark)

$A^2-5A-14I=\begin{bmatrix}29 & -25\\-20 & 24\end{bmatrix}-\begin{bmatrix}15 & -25\\-20 & 10\end{bmatrix}-\begin{bmatrix}14 & 0\\0 & 14\end{bmatrix}$

$$=\begin{bmatrix}(29-15-14) & -25+25-0\\(-20+20-0) & 24-10-14\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$ (1 mark)

9. If $2A+B=\begin{bmatrix}4 & 4 & 7\\7 & 3 & 4\end{bmatrix}$ **and** $A-2B=\begin{bmatrix}-3 & 2 & 1\\1 & -1 & 2\end{bmatrix}$**, Find A and B**

Answer: $2A+B=\begin{bmatrix}4 & 4 & 7\\7 & 3 & 4\end{bmatrix}$ …(1)

$A-2B=\begin{bmatrix}-3 & 2 & 1\\1 & -1 & 2\end{bmatrix}$ …(2)

(Redundant, hence removed)

$2A-4B=\begin{bmatrix}-6 & 4 & 2\\2 & -2 & 4\end{bmatrix}$ ...(3) (1 mark)

Subtracting, (3) from (1) $5B=\begin{bmatrix}10 & 0 & 5\\5 & 5 & 0\end{bmatrix}$ $\therefore B=\begin{bmatrix}2 & 0 & 1\\1 & 1 & 0\end{bmatrix}$ (1 mark)

$2A+B=\begin{bmatrix}4 & 4 & 7\\7 & 3 & 4\end{bmatrix}$ $\therefore 2A=\begin{bmatrix}4 & 4 & 7\\7 & 3 & 4\end{bmatrix}-\begin{bmatrix}2 & 0 & 1\\1 & 1 & 0\end{bmatrix}$

$\therefore$ $2A=\begin{bmatrix}+2 & 4 & 6\\6 & 2 & 4\end{bmatrix}$ $\therefore A=\begin{bmatrix}1 & 2 & 3\\3 & 1 & 2\end{bmatrix}$ (1 mark)

Five marks questions

10. If $A=\begin{bmatrix}1 & 3\\2 & 4\end{bmatrix}$ **and** $B=\begin{bmatrix}1 & 4\\2 & 5\end{bmatrix}$ **verify** $(AB)'=B'A'$**.**

Answer: $A\begin{bmatrix}1 & 3\\2 & 4\end{bmatrix}$, $A'=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$, $B=\begin{bmatrix}1 & 4\\2 & 5\end{bmatrix}$, $B'=\begin{bmatrix}1 & 2\\4 & 5\end{bmatrix}$ (1 mark)

$AB=\begin{bmatrix}1 & 3\\2 & 4\end{bmatrix}\begin{bmatrix}1 & 4\\2 & 5\end{bmatrix}=\begin{bmatrix}7 & 19\\10 & 28\end{bmatrix}$ (1 mark)

LHS $=(AB)'=\begin{bmatrix}7 & 10\\19 & 28\end{bmatrix}$ …(1) (1 mark)

$\text{RHS}=B'A'=\begin{bmatrix}1 & 2\\4 & 5\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$

RHS $=\begin{bmatrix}7 & 10\\19 & 28\end{bmatrix}$ …(2) (1 mark)

From (1) & (2) LHS = RHS (1 mark)

11. If $A=\begin{bmatrix}1 & 2 & 3\\3 & -2 & 1\\4 & 2 & 1\end{bmatrix}$ **show that** $A^3-23A=40I$

Answer: $A^2 = \begin{bmatrix}1 & 2 & 3\\3 & -2 & 1\\4 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\3 & -2 & 1\\4 & 2 & 1\end{bmatrix} = \begin{bmatrix}19 & 4 & 8\\1 & 12 & 8\\14 & 6 & 15\end{bmatrix}$ (1 mark)

$A^3 = A^2A = \begin{bmatrix}19 & 4 & 8\\1 & 12 & 8\\14 & 6 & 15\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\3 & -2 & 1\\4 & 2 & 1\end{bmatrix} = \begin{bmatrix}63 & 46 & 69\\69 & -6 & 23\\92 & 46 & 63\end{bmatrix}$ (1 mark)

$23A = 23\begin{bmatrix}1 & 2 & 3\\3 & -2 & 1\\4 & 2 & 1\end{bmatrix} = \begin{bmatrix}23 & 46 & 69\\69 & -46 & 23\\92 & 46 & 23\end{bmatrix}$ (1 mark)

$40I = 40\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}40 & 0 & 0\\0 & 40 & 0\\0 & 0 & 40\end{bmatrix}$ …(1) (1 mark)

$A^3 - 23A = \begin{bmatrix}63 & 46 & 69\\69 & -6 & 23\\92 & 46 & 63\end{bmatrix} - \begin{bmatrix}23 & 46 & 69\\69 & -46 & 23\\92 & 46 & 23\end{bmatrix} = \begin{bmatrix}40 & 0 & 0\\0 & 40 & 0\\0 & 0 & 40\end{bmatrix}$ …(2) (1 mark)

From $(1)\&(2)$

$\therefore A^3 - 23A = 40I$

12. If $A = \begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$, $B = \begin{bmatrix}2 & 0\\1 & 3\end{bmatrix}$ **and** $C = \begin{bmatrix}1 & 1\\2 & 3\end{bmatrix}$ **calculate** AB, AC **and** $A(B+C)$. **Verify the**

$AB + AC = A(B+C)$

Answer: $AB = \begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 3\end{bmatrix} = \begin{bmatrix}2+2 & 0+6\\4+1 & 0+3\end{bmatrix} = \begin{bmatrix}4 & 6\\5 & 3\end{bmatrix}$ (1 mark)

$AC = \begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\2 & 3\end{bmatrix} = \begin{bmatrix}1+4 & 1+6\\2+2 & 2+3\end{bmatrix} = \begin{bmatrix}5 & 7\\4 & 5\end{bmatrix}$ (1 mark)

Addition $\rightarrow AB + AC = \begin{bmatrix}9 & 13\\9 & 8\end{bmatrix}$ …(1) (1 mark)

$B + C = \begin{bmatrix}2 & 0\\1 & 3\end{bmatrix} + \begin{bmatrix}1 & 1\\2 & 3\end{bmatrix}$

$= \begin{bmatrix}3 & 1\\3 & 6\end{bmatrix}$ (1 mark)

$A(B+C) = \begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}3 & 1\\3 & 6\end{bmatrix}$

$= \begin{bmatrix}9 & 13\\9 & 8\end{bmatrix}$ …(2) (1 mark)

$\therefore$ from (1) and (2) $A(B+C) = AB + AC$