Blue Print (As per PU Board)

Topic	1 mark questions	2 marks questions	4 marks questions	5 marks questions	Total Marks
Determinants	1	1	1	1	12

One mark questions

1. **If A is a square matrix with $|A| = 6$, find the value of $|AA'|$.**

   Answer: $|AA'| = |A||A'| = 6 \times 6 = 36$. (1 mark)

2. **If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, find the value of x.**

   Answer: $x^2 - 36 = 36 - 36 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6.$ (1 mark)

3. **Find the ad joint of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$**

   Answer: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $A_{11} = 4, A_{12} = -3, A_{21} = -2, A_{22} = 1,\ Adj\ A = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix}$ (1 mark)

   $Adj\ A = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

Two marks questions

4. **Evaluate $\begin{vmatrix} 4321 & 4322 \\ 4323 & 4324 \end{vmatrix}$**

   Answer: $R_2 \to R_2 - R_1$

   $\Delta = \begin{vmatrix} 4321 & 4322 \\ 4323 - 4321 & 4324 - 4322 \end{vmatrix} = \begin{vmatrix} 4321 & 4322 \\ 2 & 2 \end{vmatrix}$ (1 mark)

   $= 2\begin{vmatrix} 4321 & 4322 \\ 1 & 1 \end{vmatrix} = 2(4321 - 4322) = 2(-1) = -2$ (1 mark)

5. **Solve $x + 3y = 5, 2x + 6y = 8$ using matrix method**

   Answer: The given equations can be written as $AX = B$ where

   $A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \end{bmatrix},\ B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ (1 mark)

   $|A| = \begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix} = 6 - 6 = 0 \quad \Rightarrow A$ is singular.

   Hence solution does not exist (1 mark)

6. **Evaluate using property of determinants $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$.**

   Answer: $C_3 \to C_3 - C_1 \Rightarrow \begin{vmatrix} 2 & 7 & 63 \\ 3 & 8 & 72 \\ 5 & 9 & 81 \end{vmatrix}$ (1 mark)

   Taking 9 as common factor from C_3.

$$\begin{vmatrix} 2 & 7 & 7 \\ 3 & 8 & 8 \\ 5 & 9 & 9 \end{vmatrix} = 0 \quad [\because C_2 = C_3]$$ (1 mark)

Four marks questions

7. **If a, b, c are in $A.P$. Show that** $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} = 0$

Answer: Since a, b, c are in $A.P$ we have $2b = a + c$ ...(1) (1 mark)

LHS= $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$ Apply $R_1 \to R_1 - 2R_2 + R_3$

$$= \begin{vmatrix} 0 & 0 & 2a-4b+2c \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$ (1 mark)

Using (1), $2a - 4b + 2c = 2(2b) - 4b = 0$

$$\therefore \text{LHS} = \begin{vmatrix} 0 & 0 & 0 \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$ (1 mark)

$\therefore$ LHS = 0 = RHS [$\therefore$ all the elements in R_1 are zeros] (1 mark)

8. **Using properties of determinant, prove that** $\begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix} = 0$

Answer: $\text{LHS} = \begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix} = \begin{vmatrix} \sin\alpha & \cos\alpha & \cos\alpha\cos\delta - \sin\alpha\sin\delta \\ \sin\beta & \cos\beta & \cos\beta\cos\delta - \sin\beta\sin\delta \\ \sin\gamma & \cos\gamma & \cos\gamma\cos\delta - \sin\gamma\sin\delta \end{vmatrix}$ (1 mark)

Apply $C_3 \to C_3 + C_1 \sin\delta - C_2 \cos\delta$ (1 mark)

$$\text{LHS} = \begin{vmatrix} \sin\alpha & \cos\alpha & 0 \\ \sin\beta & \cos\beta & 0 \\ \sin\gamma & \cos\gamma & 0 \end{vmatrix}$$ (1 mark)

LHS = 0 = RHS [$\because$ all the elements in C_3 are zeros] (1 mark)

9. **Prove that** $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

Answer: $\text{LHS} = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

Applying $R_1 \to R_1 + R_2 + R_3$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$ (1 mark)

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$\text{LHS} = (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$$ (1 mark)

$$= (a+b+c)^3 \begin{vmatrix} 1 & 0 & 0 \\ 2b & -1 & 0 \\ 2c & 0 & -1 \end{vmatrix}$$ (Taking common $(a+b+c)$ from C_1 and C_3) (1 mark)

$$\text{LHS} = (a+b+c)^3 (1\times -1 \times -1)$$

$$= (a+b+c)^3 = \text{RHS}$$ (1 mark)

Five marks questions

10. **Let** $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$ **verify that** $(AB)^{-1} = B^{-1}A^{-1}$**.**

Answer: $|A| = 1$, $Adj\ A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$ $\therefore A^{-1} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$ (1 mark)

$|B| = -2$, $Adj\ A = \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$ $\therefore B^{-1} = -\frac{1}{2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$ (1 mark)

$AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$ (1 mark)

$(AB)^{-1} = \frac{-1}{2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$ ...(1) (1 mark)

$B^{-1}A^{-1} = -\frac{1}{2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$ ...(2) (1 mark)

From (1) and (2) $(AB)^{-1} = B^{-1}A^{-1}$

11. **If** $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$**, Show that** $A^2 - 5A + 7I = 0$**, Hence find** A^{-1}**.**

Answer: $A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$, $7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$ (1 mark)

Consider $A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$. (1 mark)

$\therefore A^2 - 5A + 7I = 0 \times A^{-1}$

$A^2A^{-1} - 5AA^{-1} + 7IA^{-1} = 0$ (1 mark)

$A - 5I + 7A^{-1} = 0$

$\therefore 7A^{-1} = 5I - A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$7A^{-1} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ (1 mark)

$\therefore A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ (1 mark)

12. **If** $A = \begin{bmatrix} 2 & 3 \\ -2 & -6 \end{bmatrix}$ **verify that** $A(adj\ A) = (adj\ A)A = |A|I.$

Answer: $adj\ A = \begin{bmatrix} -6 & -3 \\ 2 & 2 \end{bmatrix}$

$|A| = -12 + 6 = -6$ (1 mark)

$|A|I = \begin{bmatrix} -6 & 0 \\ 0 & -6 \end{bmatrix}$ ...(1) (1 mark)

$A(adj\ A) = \begin{bmatrix} 2 & 3 \\ -2 & -6 \end{bmatrix}\begin{bmatrix} -6 & -3 \\ 2 & 2 \end{bmatrix}$

$= \begin{bmatrix} -6 & 0 \\ 0 & -6 \end{bmatrix}$ ...(2) (1 mark)

$(adj\ A)A = \begin{bmatrix} -6 & -3 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -2 & -6 \end{bmatrix}$

$= \begin{bmatrix} -6 & 0 \\ 0 & -6 \end{bmatrix}$ ...(3) (1 mark)

From $(1),(2)$ and (3) we get $A(adj\ A) = (adj\ A)A = |A|I$ (1 mark)