## Blue Print (As per PU Board)

| Topic | 1 mark questions | 2 marks questions | 3 marks questions | 4 marks questions | 5 marks questions | Total Marks |
|---|---|---|---|---|---|---|
| Continuity and Differentiability | 1 | 2 | 2 | 1 | 1 | 20 |

### One mark questions

**1. If $y = e^{\cos x}$, find $\frac{dy}{dx}$**

Answer: $\frac{dy}{dx} = e^{\cos x} \cdot \frac{d}{dx}(\cos x) = e^{\cos x}(-\sin x)$

2. **Differentiate the following w.r.t $x$ $\tan(2x+3)$**

Answer: $y = \tan(2x+3)$

$\therefore \frac{dy}{dx} = \sec^2(2x+3).2 = 2\sec^2(2x+3)$

3. **Examine the continuity of $f(x) = 2x+3$ at x=1**

Answer: $f(x) = 2x+3$

$\therefore f(1) = 2(1)+3 = 5 \qquad \ldots(1)$

$\lim_{x\to 1} f(x) = 2(1)+3 = 5 \qquad \ldots(2)$

from (1) & (2) $f(x)$ is continuous at $x = 1$

### Two marks questions

4. **Discuss the continuity of $f(x) = |x|$ at $x = 0$**

Answer: Let $f(x) = |x| = \begin{cases} x, if\ x \geq 0 \\ -x \text{ if x<0} \end{cases}$ (1 mark)

At $x = 0, f(0) = 0$

LHL $= \lim_{x\to 0} f(x) = \lim_{x\to 0}(-x) = \lim_{h\to 0}(-(o-h)) = 0$

RHL $= \lim_{x\to 0^+} f(x) = \lim_{x\to 0}(x) = \lim_{h\to 0}(o+h) = 0$ (1 mark)

$\therefore$ LHL=RHL $= f(0) \Rightarrow f(x)$ is continuous at $x = 0$

**5. Discuss the continuity of $f(x) = x^3 + x^2 - 1$**

Answer: $f(x)$ is defined at all real point c. $\therefore f(c) = c^3 + c^2 - 1$ (1 mark)

$\lim_{x\to c} f(c) = c^3 + c^2 - 1$

$\therefore f(c) = \lim_{x\to c} f(c)$ (1 mark)

Hence $f(x)$ is continuous

**6. Show that every polynomial function is continuous**

Answer: Let $f(x) = a_0 + a_1 x + ---- + a_{n-1}x^{n-1} + a_n x^n, a_n \neq 0$ $f(x)$ is defined at all real point c. (1 mark)

$f(c) = a_0 + a_1 c^2 + ---- + a_n c^n$

$\lim_{x\to c} f(x) = a_0 + a_1 c + a_2 c^2 + ---- + a_n c^n$

$\therefore \lim_{x \to c} f(x) = f(c)$

$\therefore f(x)$ is continuous (1 mark)

### Three marks questions

**7. If** $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, **find** $\frac{dy}{dx}$

Answer: Put $x = \tan\theta, \theta = \tan^{-1} x$ (1 mark)

$y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \cos^{-1}(\cos 2\theta)$

$y = 2\theta = 2\tan^{-1} x$ (1 mark)

Differentiating, $\frac{dy}{dx} = 2 \times \frac{1}{1+x^2} = \frac{2}{1+x^2}$ (1 mark)

**8.** $y = \sin^{-1} x + \sin^{-1}\sqrt{1-x^2}$, **find** $\frac{dy}{dx}$

Answer: put $x = \sin\theta, \theta = \sin^{-1} x$ (1 mark)

$y = \sin^{-1}(\sin\theta) + \sin^{-1}\sqrt{1-\sin^2\theta}$ (1 mark)

$= \theta + \sin^{-1}(\cos\theta)$

$= \theta + \sin^{-1}\left(\sin\left(\frac{\pi}{2} - \theta\right)\right)$

$= \theta + \frac{\pi}{2} - \theta = \frac{\pi}{2}$ (1 mark)

$\therefore \frac{dy}{dx} = 0$

**9.** $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, **find** $\frac{dy}{dx}$

Answer: put $x = \tan\theta, \theta = \tan^{-1} x$

$\therefore y = \sin^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$ (1 mark)

$y = \sin^{-1}(\cos 2\theta) = \sin^{-1}\left(\sin\left(\frac{\pi}{2} - 2\theta\right)\right)$ (1 mark)

$y = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2\tan^{-1} x$ (1 mark)

$\therefore \frac{dy}{dx} = 0 - 2 \times \frac{1}{1+x^2} = \frac{-2}{1+x^2}$

### Five marks questions

**10. If** $y = Ae^{mn} + Be^{nx}$, **then prove that** $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + (mn)y = 0$

Answer: $y = Ae^{mx} + Be^{nx}$

Differentiating, $\frac{dy}{dx} = A.me^{mx} + B.ne^{nx}$ (1 mark)

Differentiating, $\frac{d^2y}{dx} = Am^2e^{mx} + Bn^2e^{nx}$ (1 mark)

Consider $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = Am^2e^{mx} + Bn^2e^{nx} - (m+n)\left(Ame^{mx} + Bne^{nx}\right), + mn\left(Ae^{mx} + Be^{nx}\right)$ (1 mark)

$= Am^2e^{mx} + Bn^2e^{nx} - Am^2e^{mx} - Bmne^{nx} - Amne^{mx} - Bn^2e^{nx} + mnAe^{mx} + mnBe^{nx}$
$= 0$
$\therefore \frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$ (2 marks)

**11. If** $y = \left(\tan^{-1}x\right)^2$, **show that** $\left(x^2+1\right)^2 y_2 + 2x\left(x^2+1\right)y_1 = 2$

Answer: $y = \left(\tan^{-1}x\right)^2$

Differentiating, $\frac{dy}{dx} = 2\left(\tan^{-1}x\right)\frac{1}{1+x^2}$ (1 mark)

$\left(1+x^2\right)\frac{dy}{dx} = 2\tan^{-1}x$ (1 mark)

Differentiating, $\left(1+x^2\right)\ \frac{d^2y}{dx^2} + \frac{dy}{dx}(2x) = 2\frac{1}{1+x^2}$ (1 mark)

Multiplying by $\left(1+x^2\right)$,

$\left(1+x^2\right)^2\frac{d^2y}{dx^2} + \frac{dy}{dx}2x\left(1+x^2\right) = 2$
$\Rightarrow \left(x^2+1\right)^2\frac{d^2y}{dx^2} + 2x\left(1+x^2\right)y_1 = 2$ (2 mark)

**12. If** $y = 3\cos(\log x) + 4\sin(\log x)$, **show that** $x^2y_2 + xy_1 + y = 0$

Answer: $y = 3\cos(\log x) + 4\sin(\log x)$

Differentiating, $\frac{dy}{dx} = 3\left(-\sin(\log x)\frac{1}{x}\right) + 4\cos(\log x).\frac{1}{x}$ (1 mark)

$\Rightarrow x\frac{dy}{dx} = -3\sin(\log x) + 4\cos(\log x)$ (1 mark)

Differentiating, $x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{-3\cos(\log x)}{x} + 4\left(\frac{-\sin(\log x)}{x}\right)$ (1 mark)

Multiplying throughout by $x$,

$\frac{x^2d^2y}{dx^2} + x\frac{dy}{dx} = -\left[3\cos(\log x) + 4\sin(\log x)\right]$ (1 mark)

$x^2y_2 + xy_1 = -y. \Rightarrow x^2y_2 + xy_1 + y = 0$ (1 mark)