Blue Print (As per PU Board)

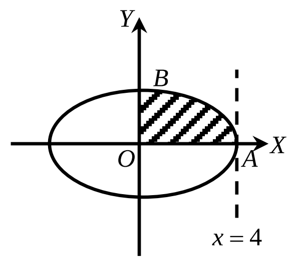
Topic	1 mark questions	2 marks questions	3 marks questions	5 marks questions	Total Marks
Application of Integrals	–	–	1	1	8

Three marks questions

1. Find the area of the region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$

Answer: Area of ellipse, $A=4$ (Area of sector AOB)

$A=4\int_0^4 y\,dx$ (1 mark)



$\frac{x^2}{16}+\frac{y^2}{9}=1 \Rightarrow \frac{y^2}{9}=1-x^2/16$

$\therefore y^2=\frac{9}{16}\left(16-x^2\right), y=\frac{3}{4}\sqrt{16-x^2}$

$A=4\left\{\frac{3}{4}\int_0^4\sqrt{16-x^2}\,dx\right\}$

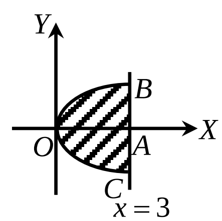
$=3\left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\Big|_0^4\right]$ (1 mark)

$=3\left[\frac{4}{2}\sqrt{16-16}+8\sin^{-1}\frac{4}{4}\right]-0$

$=24\sin^{-1}1=24\times\frac{\pi}{2}=12\pi$ sq. units (1 mark)

2. Find the area of the region bounded by the curve $y^2=4x$ **and the line** $x=3$.

Answer: Required area $A=2\int_0^3 y\,dx$ (1 mark)

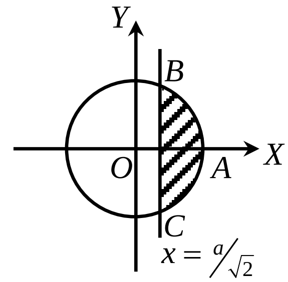


$y^2=4x, y=2\sqrt{x}$

$\therefore A=2\int_0^3 2\sqrt{x}dx=4\left[\frac{x^{3/2}}{3/2}\Big|_0^3\right]$ (1 mark)

$=\frac{8}{3}\left(3^{3/2}\right)=\frac{8}{3}\left(3\sqrt{3}\right)=8\sqrt{3}$ sq. units (1 mark)

3. Find the area of the smaller part of the circle $x^2+y^2=a^2$ cut off by the line $x=\frac{a}{\sqrt{2}}$.



Answer: Required area $=2\int_{a/\sqrt{2}}^{a} y\,dx$ (1 mark)

$x^2+y^2=a^2 \Rightarrow y=\sqrt{a^2-x^2}$

$\therefore A=2\int_{a/\sqrt{2}}^{a}\sqrt{a^2-x^2}\,dx$

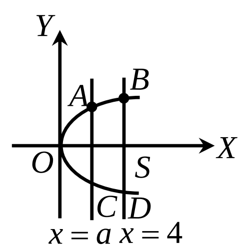
$=2\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\Big|_{a/\sqrt{2}}^{a}\right]$ (1 mark)

$A=2\left[0+\frac{a^2}{2}\left(\frac{\pi}{2}\right)-\frac{a}{2\sqrt{2}}\cdot\frac{a}{\sqrt{2}}-\frac{a^2}{2}\left(\frac{\pi}{4}\right)\right]$

$=2\left(\frac{\pi a^2}{8}-\frac{a^2}{4}\right)=\frac{a^2}{4}(\pi-2)$ sq. units (1 mark)

4. The area between $x=y^2$ and $x=4$ is divided into two equal parts by the line $x=a$, find the value of a

Answer: Given Area of $OCA=\frac{1}{2}$ area of OBD.



$2\int_0^a y\,dx=\frac{1}{2}\times 2\int_0^4 y\,dx$ (1 mark)

$y^2=x, y=\sqrt{x}$

$\therefore 2\int_0^a\sqrt{x}dx=\int_0^4\sqrt{x}dx$

$2\left[\frac{x^{3/2}}{3/2}\Big|_0^a\right]=\frac{x^{3/2}}{3/2}\Big|_0^4$ (1 mark)

$\frac{4}{3}a^{3/2}=\frac{2}{3}\left(4^{3/2}\right)$

$\frac{4}{3}a^{3/2}=\frac{16}{3}$ $\quad\therefore a^{3/2}=4$

$\therefore a=4^{2/3}$ (1 mark)

5. **Find the area of the region bounded by the curve $y^2 = 9x$, the lines $x = 2, x = 4$ and x -axis in the first quadrant.**

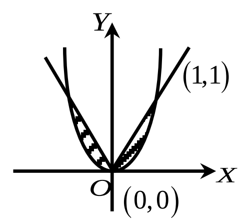
Answer: Area $A = \int_2^4 y\,dx$ (1 mark)

$y^2 = 9x, \ \therefore y = 3\sqrt{x}$

$A = \int_2^4 3\sqrt{x}\ dx = 3\left.\frac{x^{3/2}}{3/2}\right|_2^4$ (1 mark)

$= 2\left(4^{3/2} - 2^{3/2}\right) = 2\left(8 - 2\sqrt{2}\right)$ sq. units (1 mark)

6. **Find the area of the region bounded by the parabola $y = x^2$** and $y = |x|$



Answer: $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

The curve is symmetrical about $y -$ axis.

when $x \geq 0, |x| = x, \ \therefore y = x, \ x = x^2, \ x^2 - x = 0, \ x(x - x) = 0, \ x = 0, x = 1$

$y = 0, y = 1$ $(1,1)$ and $(0,0)$ are point of intersection

Required area $= 2\left[\int_0^1 |x|\,dx - \int_0^1 x^2 dx\right]$ (1 mark)

$= 2\left[\int_0^1 x\,dx - \int_0^1 x^2 dx\right]$

$= 2\left[\left.\frac{x^2}{2} - \frac{x^3}{3}\right|_0^1\right]$ (1 mark)

$A = 2\left[\frac{1}{2} - \frac{1}{3}\right] = 2\left(\frac{1}{6}\right) = \frac{1}{3}$ sq. units (1 mark)

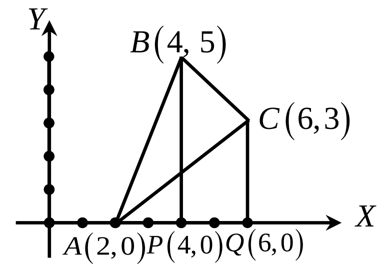
Five marks questions

7. **Using method of integration, find the area of the triangle ABC, whose coordinates of its vertices are $A(2,0), B(4,5), C(6,3)$.**

Answer: We have $A(2,0), B(4,5)$

Equation of AB, $y - 0 = \left(\frac{5-0}{4-2}\right)(x - 2)$ using $\left[y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)\right]$

$\Rightarrow y=\frac{5}{2}(x-2)$ ...(1)



Equation of $BC, y-5=\frac{3-5}{6-4}(x-4)$

$\Rightarrow y=9-x$ ...(2) (1 mark)

Equation of $AC, y-0=\frac{3-0}{6-2}(x-2)$

$y=\frac{3}{4}(x-2)$ ...(3) Fig. (1 mark)

Area of triangle ABC = Area of triangle ABP + Area of trapezium $BCPQ$ − area of ΔACQ

= Area under AB + Area under BC − area under AC

$$=\int_2^4 \frac{5}{4}(x-2)\,dx+\int_4^6 (9-x)\,dx-\int_2^6 \frac{3}{4}(x-2)\,dx$$ (1 mark)

$$=\frac{5}{2}\left[\frac{x^2}{2}-2x\Big|_2^4\right]+\left(9x-\frac{x^2}{2}\Big|_4^6\right)-\frac{3}{4}\left[\frac{x^2}{2}-2x\Big|_2^6\right]$$ (1 mark)

$$=\frac{5}{2}\left[\left(\frac{16}{2}-8\right)-\left(\frac{4}{2}-4\right)\right]+\left[9(6-4)-\frac{1}{2}(36-16)\right]-\frac{3}{4}\left[\left(\frac{36}{2}-\frac{4}{2}\right)-2(6-2)\right]$$

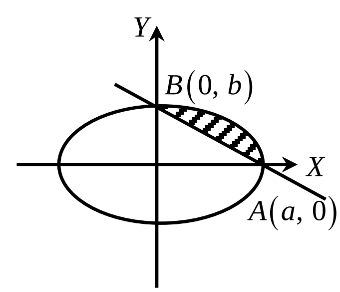
$$=\frac{5}{2}(0+2)+(18-10)-\frac{3}{4}(8)$$

$=5+8-6=7$ sq. units (1 mark)

8. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$ (F)

Answer: Required area = Shaded region. $=\int_0^a (A_1-A_2)\,dx$ where A_1 = Area bounded by the ellipse and A_2 = Area bounded by the straight line (1 mark)

$$A=\int_0^a \frac{b}{a}\sqrt{a^2-x^2}\,dx-\int_0^a \frac{b}{a}(a-x)\,dx$$ (1 mark)



$$=\frac{b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\Big|_0^a\right]-\frac{b}{a}\left[ax-\frac{x^2}{2}\Big|_0^a\right]$$ (1 mark)

$$=\frac{b}{a}\left[0+\frac{a^2}{2}\sin^{-1}\frac{a}{a}\right]-\frac{b}{a}\left(a^2-\frac{a^2}{2}\right)$$

$$=\frac{b}{a}\left(\frac{a^2}{2}\times\frac{\pi}{2}\right)-\frac{b}{a}\left(\frac{a^2}{2}\right)$$ Fig. (1 mark)

$= \frac{\pi ab}{4} - \frac{ab}{2} = \frac{ab}{4}(\pi - 2)$ (1 mark)

9. Find the area bounded by the Curves $y = x^2 + 2, y = x, x = 0$ and $x = 3$

Answer: $y = x$ is a straight line passing through the origin which lies below the parabola $y = x^2 + 2$

The required area is the shaded region

Area=Area under the parabola-Area under the line $y = x$ (1 mark)

$= \int_0^3 (x^2 + 2)dx - \int_0^3 x dx$ (1mark)

$= \left(\frac{x^3}{3} + 2x\right)\Big|_0^3 - \frac{x^2}{2}\Big|_0^3$ (1mark)

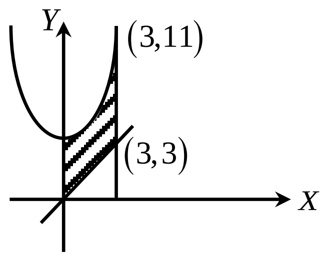


Fig. (1mark)

$= (9+6) - (0+0) - \frac{9}{2}$

$= \frac{21}{2}$ sq. units (1mark)

10. Find the area bounded by the curve $y = \cos x$ **between** $x = 0$ **and** $x = 2\pi$

Answer: Required Area = Area of OABO + Area of BCDB + Area of DFED (1mark)

$= \int_0^{\frac{\pi}{2}} \cos x dx + \left|\int_{\frac{\pi}{2}}^{3\frac{\pi}{2}} \cos x dx\right| + \int_{3\frac{\pi}{2}}^{2\pi} \cos x dx$ (1mark)

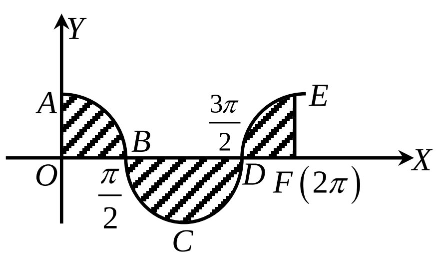


Fig. (1 mark)

$A = \sin x\Big|_0^{\frac{\pi}{2}} + \left|\sin x\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\right| + \sin x\Big|_{3\frac{\pi}{2}}^{2\pi}$ (1 mark)

$= \left(\sin\frac{\pi}{2} - \sin 0\right) + \left|\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right| + \left(\sin 2\pi - \sin\frac{3\pi}{2}\right)$

$= 1 + 2 + 1 = 4$ sq. units (1 mark)

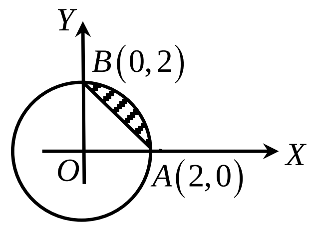
11. Find the smaller area enclosed by the circle $x^2 + y^2 = 4$ **and the line** $x + y = 2$

Answer: Required area=area of circle- area bounded by line (1mark)

Solving $(x^2 + y^2 = 4)$ and $x + y = 2$ we get $x = 2, x = 0, y = 0, y = 2$ (1mark)

$\therefore A=\int_0^2 \sqrt{4-x^2}\,dx-\int_0^2 (2-x)\,dx$ (1mark)

$A=\left(\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right)\Bigg|_0^2-\left(2x-\frac{x^2}{2}\Bigg|_0^2\right)$ (1mark)



$=\left(0+2\sin^{-1}1\right)-0-(4-2)-(0)$

$=2\left(\frac{\pi}{2}\right)-2=(\pi-2)$ sq units (1mark)

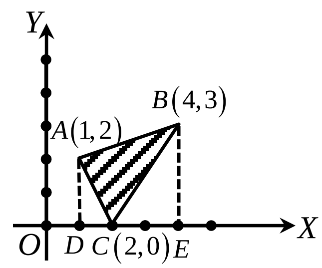
12. Using the method of integration find the area of the region bounded by the lines $2x+y=4, 3x-2y=6, x-3y+5=0$

Answer: We have

$AC: \ 2x+y=4 \quad \ldots(1),$

$BC: \ 3x-2y=6 \quad \ldots(2),$ Fig. (1 mark)

$AB: \ x-3y+5=0 \quad \ldots(3)$



Solving (1) & (2) we get $x=2, y=0$

Solving (2) & (3) we get $x=4, y=3$ (1 mark)

Solving (3) & (1) we get $x=1, y=2$

Required Area = Area of $ADEB-$ area of $ADC-$ area of BCE (1 mark)

$=$Area bounded by $x-3y+5-$ area bounded by $2x+y=4-$ area bounded by $3x-2y=6$

$=\int\limits_1^4 \frac{x+5}{3}dx-\int\limits_1^2 (4-2x)\,dx-\int\limits_2^4 \frac{3x-6}{2}dx$

$=\frac{1}{3}\left(\frac{x^2}{2}+5x\right)\Bigg|_1^4-\left(4x-x^2\right)\Big|_1^2-\frac{1}{2}\left(\frac{3x^2}{2}-6x\right)\Bigg|_2^4$ (1 mark)

$=\frac{1}{3}\left[(8+20)-\left(\frac{1}{2}+5\right)\right]-\left[(8-4)-(4-1)\right]-\frac{1}{2}\left[(24-24)-(6-12)\right]$

$=\frac{1}{3}\left[28-\frac{1}{2}-5\right]-(4-3)-\frac{1}{2}(0+6)$

$=\frac{15}{2}-1-3=\frac{7}{2}$ sq. units (1 mark)