**Blue Print (As per PU Board)**

| Topic | 1 mark questions | 2 marks questions | 3 marks questions | 5 marks questions | Total Marks |
|---|---|---|---|---|---|
| Differential Equation | – | 1 | 1 | 1 | 10 |

### Two marks questions

1. **Find the order and degree of the following differential equation** $\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$

   Answer: order 2 and degree 1

2. **Verify whether in the following problems, given function is a solution of the corresponding differential equation** $y=x\sin 3x$ $\quad \frac{d^2y}{dx^2}+9y-6\cos 3x=0$

   Answer: $y=x\sin 3x$

   Differentiating , $\frac{dy}{dx}=x.3\cos 3x+\sin 3x$

   $\frac{d^2y}{dx^2}=-9x\sin 3x+3\cos 3x+3\cos 3x$

   $=6\cos 3x-9x\sin 3x$ (1 mark)

   Consider $\frac{d^2y}{dx^2}+9y-6\cos 3x=6\cos 3x-9x\sin 3x+9x\sin 3x-6\cos 3x=0$ (1 mark)

   Hence, $y=x\sin 3x$ is a solution

3. **Form the differential equation representing the family of curves** $y=mx$ **where m is arbitrary constant**

   Answer: we have $y=mx \quad \ldots(1)$

   Differentiating wrt $x$, $\frac{dy}{dx}=m$ (1 mark)

   Substituting this in $(1)$for $m\}$ we get $y=\left(\frac{dy}{dx}\right)x \Rightarrow x\frac{dy}{dx}-y=0$ (1 mark)

4. **Find the general solution of** $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$

   Answer: separating the variables, $\frac{dy}{1+y^2}=\frac{dx}{1+x^2}$ (1 mark)

   Integrating, $\int\frac{dy}{1+y^2}=\int\frac{dx}{1+x^2}+c$

   $\tan^{-1}y=\tan^{-1}x+c$ (1 mark)

### Three marks questions

5. **Find the general solution of** $\sec^2 x\tan y dx+\sec^2 y\tan x dy=0$

   Answer: $\sec^2 x\tan y dx+\sec^2 y\tan x\, dy=0 \quad \cdots(1)$

   To separate the variables $\div(1)$ by $\tan y\tan x$

   $\therefore\frac{\sec^2 x}{\tan x}dx+\frac{\sec^2 y}{\tan y}dy=0$ (1 mark)

   Integrating, $\int\frac{\sec^2 x}{\tan x}dx+\int\frac{\sec^2 y}{\tan y}dy=0$ (1 mark)

$\log|\tan x| + \log|\tan y| = c$ (1 mark)

6. **Form the differential equation representing the family of curves $y = a\sin(x+b)$ where a,b are arbitrary constants**

Answer: consider $y = a\sin(x+b)$

Differentiating, $\frac{dy}{dx} = a\cos(x+b)$ (1 mark)

Again differentiating, $\frac{d^2y}{dx^2} = -a\sin(x+b)$ (1 mark)

$\frac{d^2y}{dx^2} = -y \qquad [\because a\sin(x+b) = y]$

$\Rightarrow \frac{d^2y}{dx^2} + y = 0$ (1 mark)

7. **Form the differential equation representing the family of parabolas having vertex at origin and axis along positive direction of y axis**

Answer: The equation of required parabola is $x^2 = 4ay$

$\therefore \frac{x^2}{y} = 4a$ (1 mark)

Differentiating wrt $x$, $\frac{y(2x) - x^2\frac{dy}{dx}}{y^2} = 0$ (1 mark)

$\therefore 2xy - x^2\frac{dy}{dx} = 0$

$\therefore x^2\frac{dy}{dx} = 2xy$

or $x\frac{dy}{dx} - 2y = 0$ (1 mark)

8. **Form the differential equation of the family of circles having centre on $y$ axis and radius 3 units**

Answer: Let the equation of the family circles be $(x)^2 + (y-a)^2 = 9$ …(1)

Differentiating wrt $x$, $2x + 2(y-a)\frac{dy}{dx} = 0$ (1 mark)

$(y-a) = \frac{-x}{\left(\frac{dy}{dx}\right)}$ (1 mark)

$(1) \Rightarrow x^2 + \frac{x^2}{\left(\frac{dy}{dx}\right)^2} = 9$

$\Rightarrow x^2(y')^2 + x^2 = 9(y')^2$

$(9 - x^2)(y')^2 = x^2$

$\therefore (x^2 - 9)(y')^2 + x^2 = 0$ (1 mark)

**Five marks questions**

9. **Find the particular solution of the differential equation**

$\frac{dy}{dx}-3y\cot x=\sin 2x;\ \ y=2 \text{ when } x=\frac{\pi}{2}$

Answer: $\frac{dy}{dx}-3\cot x.y=\sin 2x$

This is of the form, $\frac{dy}{dx}+P(x)y=Q(x)$ where $P(x)=-3\cot x, Q(x)=\sin 2x$ (1 mark)

I.F.$=e^{\int Pdx}=e^{\int -3\cot x.dx}=e^{-3\log\sin x}=e^{\log(\sin x)^{-3}}=\frac{1}{\sin^3 x}$ (1 mark)

Solution is $y(I.F)=\int(I.F)Qdx+c$

$y\left(\frac{1}{\sin^3 x}\right)=\int\frac{1}{\sin^3 x}\sin 2xdx+c$ (1 mark)

$=\int\frac{2\sin x\cos x}{\sin^3 x}dx+c$

$\frac{y}{\sin^3 x}=2\int\frac{\cos x}{\sin^2 x}dx+c$

$=2\int\frac{dt}{t^2}+c \quad \text{where } t=\sin x, dt=\cos xdx$

$=2\left(\frac{-1}{t}\right)+c$

$\frac{y}{\sin^3 x}=\frac{-2}{\sin x}+c$ (1 mark)

When $x=\frac{\pi}{2},\quad y=2$

$\frac{2}{1^3}=\frac{-2}{1}+c \qquad \therefore c=4$

$\therefore$ solution is $y\cos ec^3x=-2\cos ecx+4$ (1 mark)

10. **Find the general solution of the differential equation** $x\frac{dy}{dx}+2y=x^2\log x$

Answer: $x\frac{dy}{dx}+2y=x^2\log x$

$\div x, \frac{dy}{dx}+\frac{2}{x}y=x\log x.$ is of the form $\frac{dy}{dx}+P(x)y=Q(x)$

Where $P(x)=\frac{2}{x}, Q(x)=x\log x.$ which is linear (1 mark)

$I.F=e^{\int pdx}=e^{\int\frac{2}{x}dx}=e^{2\log|x|}=e^{\log|x|^2}=x^2$ (1 mark)

Solution is $y(I.F)=\int(I.F)qdx+c$

$y.(x^2)=\int(I.F)Qdx+c$

$y.(x^2)=\int x^2.x\log xdx+c$ (1 mark)

$x^2y=\int x^3\log xdx+c$

$x^2y=\log x\int x^3dx-\int\left[\frac{d}{dx}\log x.\int x^3dx\right]dx+c$ (1 mark)

$$x^2y = \log\cdot\frac{x^4}{4} - \int\frac{x^4}{4}\left(\frac{1}{x}\right)dx + c$$

$$x^2y = \log x\frac{x^4}{4} - \frac{x^4}{16} + c$$

$$y = \frac{x^2}{16}(4\log|x|-1) + cx^{-2}$$

$$x^2y = \log .x\frac{x^4}{4} + \frac{x^4}{16} + c$$ (1 mark)

11. **Find the particular solution of the differential equation** $\left(1+e^{2x}\right)dy + \left(1+y^2\right)e^x dx = 0$ **given that** $y=1$ **when** $x=0$

Answer: $\left(1+e^{2x}\right)dy + \left(1+y^2\right)e^x dx = 0$

$$\frac{dy}{1+y^2} + \frac{e^x}{1+e^{2x}}dx = 0$$ (separating the variables) (1 mark)

Integrating, $\int\frac{dy}{1+y^2} + \int\frac{e^x}{1+e^{2x}}dx = c$ (1 mark)

$\tan^{-1}y + \int\frac{dt}{1+t^2} = c$ where $t = e^x,\ dt = e^x dx$

$\tan^{-1}y + \tan^{-1}t = c$

$\tan^{-1}y + \tan^{-1}\left(e^x\right) = c$ (1 mark)

When, $x=0,\quad y=1$

$\tan^{-1}1 + \tan^{-1}e^0 = c \quad \Rightarrow \tan^{-1}1 + \tan^{-1}1 = c$

$2\frac{\pi}{4} = C \ \therefore C = \frac{\pi}{2}$ (1 mark)

Particular solution, $\tan^{-1}y + \tan^{-1}e^x = \frac{\pi}{2}$ (1 mark)

12. **Find a particular solution of the differential equation** $(x-y)(dx+dy) = dx-dy$ **.**
**Given that** $y=1$**, when** $x=0$

Answer: The given equation is, $(x-y)(dx+dy) = dx-dy$

$(x-y)dx + (x-y)dy - dx + dy = 0$

$(x-y-1)dx + (x-y+1)dy = 0$

$\therefore\frac{dy}{dx} = -\frac{(x-y-1)}{(x-y+1)} \quad \cdots(1)$ (1 mark)

Put $x-y=z \quad \Rightarrow 1-\frac{dy}{dx} = \frac{dz}{dx}$ (1 mark)

$(1)\Rightarrow \quad 1-\frac{dz}{dx} = -\left(\frac{z-1}{z+1}\right)$

$\frac{dz}{dx} = 1+\frac{z-1}{z+1}$

$\left(\frac{z+1}{z}\right)dz = 2dx$ (separating the variable) (1 mark)

Integrating, $\int\left(1+\frac{1}{z}\right)dz = 2x + c$

$z+\log|z|=2x+c$

$(x-y)+\log|x-y|=2x+c$ (1 mark)

When $x=0, y=-1$

$(0+1)+\log|1|=2(0)+c$

$c=1$

Solution is $(x-y)+\log|x-y|=2x+1$

$\log|x-y|=x+y+1$ (1 mark)