**Blue Print (As per PU Board)**

| Topic | 1 mark questions | 2 marks questions | 3 marks questions | 5 marks questions | Total Marks |
|---|---|---|---|---|---|
| Vector Algebra | 1 | 2 | 2 | - | 11 |

## One mark questions

**1. Find the value of $\lambda$ if $\frac{2}{3}\hat{i}-\lambda\hat{j}+\frac{2}{3}\hat{k}$ is a unit vector.**

Answer: $\vec{a}=\frac{2}{3}\hat{i}-\lambda\hat{j}+\frac{2}{3}\hat{k}$

$|\vec{a}|=1\Rightarrow\sqrt{\left(\frac{2}{3}\right)^2+\left(-\lambda^2\right)+\left(\frac{2}{3}\right)^2}=1\Rightarrow\sqrt{\lambda^2+\frac{8}{9}}=1$

$\lambda^2=1-\frac{8}{9}=\frac{1}{9}\qquad\therefore\lambda=\pm\frac{1}{3}$

**2. Find the vector joining two points $A(2,3,0),B(-1,-2,4)$**

Answer: $\overrightarrow{AB}=(-1,-2,4)-(2,3,0)=(-3,-5,4)$ or $\overrightarrow{AB}=-3\hat{i}-5\hat{j}+4\hat{k}$

**3. If $\vec{a}=(1,-2,1),\vec{b}=(-2,4,5),\vec{c}=(1,-6,-7)$ find $\vec{a}+\vec{b}+\vec{c}$**

Answer: $\vec{a}+\vec{b}+\vec{c}=\left((1+(-2))+1,(-2+4-6),(1+5-7)\right)=(0,-4,-1)$ or $\vec{a}+\vec{b}+\vec{c}=-4j-k$

**4. If the vectors $2\hat{i}+3\hat{j}-6\hat{k}$ and $4\hat{i}-m\hat{j}-12\hat{k}$ are parallel, find $m$**

Answer: Since 2 vectors are parallel, $\frac{2}{4}=\frac{3}{-m}=\frac{-6}{-12}$

$\therefore\frac{3}{-m}=\frac{1}{2}\Rightarrow m=-6$

## Two marks questions

**5. Find the angle between the vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a}\cdot\vec{b}=\sqrt{6}$**

Answer: $\cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$ (1 mark)

$=\frac{\sqrt{6}}{\sqrt{3}\cdot 2}=\frac{\sqrt{3}\cdot\sqrt{2}}{\sqrt{3}\cdot\sqrt{2}\cdot\sqrt{2}}=\frac{1}{\sqrt{2}}\qquad\therefore\theta=\cos^{-1}\frac{1}{\sqrt{2}}=\frac{\pi}{4}$ (1 mark)

**6. Find a vector in the direction of a vector $\vec{a}=\hat{i}-2\hat{j}$ that has magnitude 7 units.**

Answer: Required vector $=7\hat{a}$ where $\hat{a}=\frac{\vec{a}}{|\vec{a}|}$ (1 mark)

$\frac{7\left(\hat{i}-2\hat{j}\right)}{\sqrt{1^2+2^2}}=\frac{7\hat{i}-14\hat{j}}{\sqrt{5}}=\frac{7}{\sqrt{5}}\hat{i}-\frac{14}{5}\hat{j}$ (1 mark)

**7. Evaluate $(3\vec{a}-5\vec{b})\cdot(2\vec{a}+7\vec{b})$**

Answer: $(3\vec{a}-5\vec{b})\cdot(2\vec{a}+7\vec{b})=6(\vec{a}\cdot\vec{a})-10(\vec{b}\cdot\vec{a})+21(\vec{a}\cdot\vec{b})-35(\vec{b}\cdot\vec{b})$ (1 mark)

$=6|\vec{a}|^2-11(\vec{a}\cdot\vec{b})-35|\vec{b}|^2\quad\left[\because\vec{b}\cdot\vec{a}=\vec{a}\cdot\vec{b}\right]$ (1 mark)

8. **Show that $|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$ for any two nonzero vectors $\vec{a}$ and $\vec{b}$.**

Answer: $\left(|\vec{a}|\vec{b}+|\vec{b}|\vec{a}\right)\cdot\left(|\vec{a}|\vec{b}-|\vec{b}|\vec{a}\right)$

$=|\vec{a}|^2\left(\vec{b}\cdot\vec{b}\right)-|\vec{a}||\vec{b}|\left(\vec{b}\cdot\vec{a}\right)+|\vec{b}||\vec{a}|\left(\vec{a}\cdot\vec{b}\right)-|\vec{b}|^2\left(\vec{a}\cdot\vec{a}\right)$ (1 mark)

$=|\vec{a}|^2|\vec{b}|^2-|\vec{a}||\vec{b}|\left(\vec{a}\cdot\vec{b}\right)+|\vec{a}||\vec{b}|\left(\vec{a}\cdot\vec{b}\right)-|\vec{b}|^2|\vec{a}|^2=0$ (1 mark)

$\therefore$ The two given vectors are perpendicular to each other.

### Three marks questions

9. **If $\vec{a}=5\hat{i}-\hat{j}-3\hat{k}$ and $\vec{b}=\hat{i}+3\hat{j}-5\hat{k}$, then show that the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular to each other.**

Answer: $\vec{a}+\vec{b}=\left(5\hat{i}-\hat{j}-3\hat{k}\right)+\left(\hat{i}+3\hat{j}-5\hat{k}\right)=6\hat{i}+2\hat{j}-8\hat{k}$ (1 mark)

$\vec{a}-\vec{b}=5\hat{i}-\hat{j}-3\hat{k}-\left(\hat{i}+3\hat{j}-5\hat{k}\right)=4\hat{i}-4\hat{j}+2\hat{k}$ (1 mark)

If $\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right)=0$ then $\left(\vec{a}+\vec{b}\right)$ perpendicular $\left(\vec{a}-\vec{b}\right)$

Consider $\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right)=\left(6\times4+(-4)\times2+(-8)\times2\right)=24-8-16=0$ (1 mark)

$\therefore\left(\vec{a}+\vec{b}\right)$ is perpendicular to $\left(\vec{a}-\vec{b}\right)$

10. **Find a unit vector perpendicular to each of vector $\left(\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-\vec{b}\right)$ where $\vec{a}=\hat{i}+\hat{j}+\hat{k},\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$**

Answer: Let $\vec{c}=\vec{a}+\vec{b}$ and $\vec{d}=\vec{a}-\vec{b}$

$\vec{c}=\hat{i}+\hat{j}+\hat{k}+\hat{i}+2\hat{j}+3\hat{k}=2\hat{i}+3\hat{j}+4\hat{k}$

$\vec{d}=\hat{i}+\hat{j}+\hat{k}-\hat{i}-2\hat{j}-3\hat{k}=-\hat{j}-2\hat{k}$ (1 mark)

Unit vector perpendicular to both $\vec{c}$ and $\vec{d}=\dfrac{\vec{c}\times\vec{d}}{\left|\vec{c}\times\vec{d}\right|}$

$$\vec{c}\times\vec{d}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&3&4\\0&-1&-2\end{vmatrix}=\hat{i}(-6+4)-\hat{j}(-4-0)+\hat{k}(-2-0)=-2\hat{i}+4\hat{j}-2\hat{k}$$ (1 mark)

$\therefore\left|\vec{c}\times\vec{d}\right|=\sqrt{(-2)^2+4^2+(-2)^2}=\sqrt{4+16+4}=\sqrt{24}$

$\therefore\hat{n}=\dfrac{-2i+4j-2k}{\sqrt{24}}=\dfrac{-i+2j-k}{\sqrt{6}}$ (1 mark)

11. **Show that the vectors $\vec{a},\vec{b}$ and $\vec{c}$ are coplanar if $\vec{a}+\vec{b},\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar.**

Answer: Since $\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}$ are coplanar we have $\left[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}\right]=0$ (1 mark)

$\left(\vec{a}+\vec{b}\right)\cdot\left\{\left(\vec{b}+\vec{c}\right)\times\left(\vec{c}+\vec{a}\right)\right\}=0$

$\left(\vec{a}+\vec{b}\right)\cdot\left\{\vec{b}\times\vec{c}+\vec{c}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a}\right\}=0$ (1 mark)

$\vec{a}\cdot\vec{b}\times\vec{c}+\vec{a}\cdot\vec{b}\times\vec{a}+\vec{a}\cdot\vec{c}\times\vec{a}+\vec{b}\cdot\vec{b}\times\vec{c}+\vec{b}\cdot\vec{b}\times\vec{a}+\vec{b}\cdot\vec{c}\times\vec{a}=0$

$\vec{a}\cdot\vec{b}\times\vec{c}+0+0+0+0+\vec{b}\cdot\vec{c}\times\vec{a}=0$

$\Rightarrow\left[\vec{a}\,\vec{b}\,\vec{c}\right]+\left[\vec{b}\,\vec{c}\,\vec{a}\right]=0\Rightarrow\left[\vec{a}\,\vec{b}\,\vec{c}\right]+\left[\vec{a}\,\vec{b}\,\vec{c}\right]=0$

$\therefore 2\left[\vec{a}\,\vec{b}\,\vec{c}\right]=0$

$\therefore \left[\vec{a}\,\vec{b}\,\vec{c}\right]=0$ (1 mark)

$\therefore \vec{a}\,\vec{b}\,\vec{c}$ are coplanar.

**12. Find $x$ such that the four points $A(3,2,1)\,B(4,x,5)\,C(4,2,-2)$ and $D(6,5,-1)$ are coplanar.**

Answer: $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=(1,x-2,4)$

$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=(1,0,-3)$

$\overrightarrow{AD}=\overrightarrow{OD}-\overrightarrow{OA}=(3,3,-2)$ (1 mark)

For coplanar $\overrightarrow{AB}\cdot\left(\overrightarrow{AC}\times\overrightarrow{AD}\right)=0$

$$\begin{vmatrix}1 & x-2 & 4\\ 1 & 0 & -3\\ 3 & 3 & -2\end{vmatrix}=0$$ (1 mark)

$1(0+9)-(x-2)(-2+9)+4(3-0)=0$

$9-7x+14+12=0 \Rightarrow 7x=35$

$\therefore x=5$ (1 mark)