Blue Print (As per PU Board)

Topic	1 mark questions	2 marks questions	3 marks questions	5 marks questions	Total Marks
Wave Optics	–	–	1	–	3

One mark questions

1. **Define the term wave front?**
   Answer: A wave front is defined as the locus of all points (particles) in a medium which are vibrating in the same phase.
2. **Which type of wave front is received if light is coming from A very distant source**
   Answer: plane wave front
3. **What are coherent sources?**
   Answer: Two sources are said to be coherent when the waves emanated from them have the same frequency and a constant phase difference.

Two marks questions

4. **Two independent sources of light cannot be considered as coherent sources. Explain why**?
   Answer: Each source of light consists of a large number of atoms and light is emitted by these atoms independently and randomly. There will be no constant phase relation between the two sources. (1 mark)
   The phase difference between the two sources is rapidly changing with time. Thus two independent sources cannot be considered as coherent sources. (1 mark)
5. **Diffraction is observed in the case of sound waves while it is not easily observed for light waves. Why? Explain**
   Answer: Diffraction effects are observable only when the size of the obstacle is comparable to the wavelength of the waves. In the case of sound waves, the wavelength of sound waves is comparable to the size of the obstacle or aperture. So diffraction effects are easily observable. But in the case of light waves, the wavelength of light is extremely small compared to the size of the obstacle or object. So diffraction effects in the case of light waves are not easily observed. (1+1 mark)
6. **(a) Explain why longitudinal waves cannot be polarised.**
   Answer: In polarisation, vibrations (electric field vectors) perpendicular to the direction of propagation are restricted to only one direction. (1 mark)
   But in longitudinal waves, vibrations occur only along the direction of propagation but not perpendicular to it. Hence polarisation of longitudinal waves is not possible. (1 mark)

Three marks questions

7. **(a) Write an expression for limit of resolution and resolving power of a microscope.**
   (b) Explain how the resolving power of a microscope can be increased.
   Answer: (a) Limit of resolution a microscope

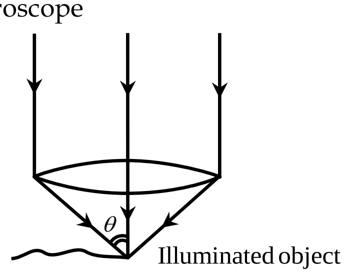
$$dx = \frac{\lambda}{2n\sin\theta}$$

(1 mark)

Resolving power

$$rp = \frac{2n\sin\theta}{\lambda}$$

(1 mark)



$\theta =$ Semi vertical angle of the cone of incident rays subtended at the objective

$n =$ Refractive index of the medium surrounding the objective.

$\lambda =$ Wavelength of the incident light

(b) The resolving power of a microscope can be increased by (i) increasing the angle θ, (ii) by decreasing the wavelength of light. (1 mark)

8. State and explain Huygens'principle of wave fronts. Derive Snell's law of refraction using Huygens'principle of wave fronts

Answer: Consider a plane surface XY that separates a denser medium of refractive index n from a rarer medium. If v_1 is the velocity of light in the rarer medium and v_2 is the velocity of light in the denser medium, then by definition,

$$n = \frac{v_1}{v_2} \quad \cdots(1)$$

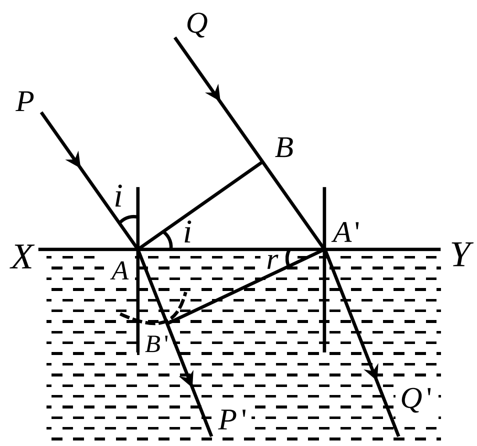


fig. (1 mark)

Consider two incident rays PA and QA′ which are incident on this surface. Draw AB perpendicular to QA[1]. AB represents the incident wave front. According to Huygens' principle, every point on the primary wavefront AB acts as a secondary source of disturbance. Let the secondary wavelet strike the surface XY at A'. The secondary wavefront from B travels a distance $v_1 t$ in time t seconds.

$\therefore\ BA' = v_1 t \quad ...(1)$ (1 mark)

The secondary wavelet from A travels a distance $v_2 t$ in the same interval of time. So with A as centre draw an arc of radius $v_2 t$. From A′ draw a tangent plane which touches this spherical arc at B'. Then $A'B'$ will be perpendicular to AB'. So A′B′ is the refracted wavefront. So the ray AP' perpendicular to this refracted wavefront AP' represents the refracted ray at A and $A'Q'$ represents the refracted ray at A'.

In $\Delta ABA', \underline{|BAA'} = i$

$$\sin i = \frac{BA'}{AA'} ......(1)$$

In $\Delta AA'B'$, $\underline{|BA'A} = r$

$$\sin r = \frac{AB'}{AA'} \quad \cdots(2)$$

Dividing (1) by (2) we get

$$\frac{\sin i}{\sin r} = \frac{\left(\frac{BA'}{AA'}\right)}{\left(\frac{AB'}{AA'}\right)} = \frac{BA'}{AB'} = \frac{v_1 t}{v_2 t} = \frac{v_1}{v_2} \qquad \therefore\ n = \frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

(1 mark)

Thus the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for a given pair of media for a given colour of light.

9. State and explain Huygens' principle of wave fronts. Show that the angle of incidence is equal to the angle of reflection in the case of reflection of light by a plane mirror, using Huygens' principle.

Answer: Consider a plane wave front AB striking a plane reflecting surface MN at an angle of incidence i. Let v be the speed of light in the medium. In a time t, the wave travels from B to C a distance BC = vt. In order to construct the reflected wave front we draw a sphere of radius vt from the point A as shown in the figure. Let CE represent the tangent plane drawn from the point C to this sphere.

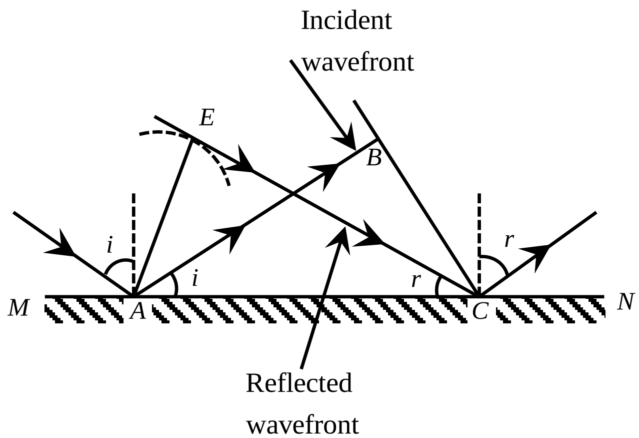


Fig. (1 mark)

AE = BC = vt (1 mark)

Compare triangles EAC and BAC

$\lfloor \underline{AEC} = \lfloor \underline{ABC} = 90°$

Side AC is common

AE = BC,

So triangles are congruent

$\therefore \lfloor \underline{B}AC = \lfloor \underline{E}CA$ $\therefore i = r$

Hence the laws of reflection are proved (1 mark)

Five marks questions

10. Give the theory of interference of light and obtain the condition for constructive and destructive interference

Answer: Consider two waves having amplitude a_1 and a_2, coming out from a source of light of wavelength λ. They have a constant phase difference ϕ between them. They are represented by

$y_1 = a_1 \sin \omega t$ …(1) and

$y_2 = a_2 \sin(\omega t + \phi)$ ... (2) respectively.

y_1 and y_2 are the particle displacements produced by the respective waves at any instant of time t.

These two waves are made to overlap. The resultant displacement produced by the two waves on superposition is given by

$y = y_1 + y_2$ (by the principle of superposition of waves) (1 mark)

$y = a_1 \sin \omega t + a_2 \sin(\omega t + \phi)$

$= a_1 \sin \omega t + a_2 [\sin \omega t \cos \phi + \cos \omega t \sin \phi]$ ... (3)

$= (a_1 + a_2 \cos \phi) \sin \omega t + (a_2 \sin \phi) \cos \omega t$

Put $(a_1 + a_2 \cos \phi) = A \cos \theta$ ... (4)

$a_2 \sin \phi = A \sin \theta$ ... (5)

Substituting these in (3), we get $y = \sin \omega t [A \cos \theta] + \cos \omega t [A \sin \theta]$

$y = \sin \omega t [A \cos \theta] + \cos \omega t [A \sin \theta]$

$y = A \sin(\omega t + \theta)$ … (6) (1 mark)

This is the equation of the combined wave formed by the superposition of the waves represented by (1) and (2). This equation is similar to the equation of the component waves. Hence A represents the resultant amplitude of the combined wave. We know that a wave carries energy. The intensity of a

wave is the energy transmitted by a wave per unit area per unit time. Let I_1 and I_2 represent the intensity of the first and second waves respectively. Let I represent the intensity of the combined wave. The intensity of a wave is directly proportional to the square of the amplitude.
Then $I_1 \propto a_1^2$, $I_2 \propto a_2^2$ and $I \propto A^2$.
Square and add (4) and (5)

$$(A\cos\theta)^2 + (A\sin\theta)^2 = (a_1 + a_2\cos\phi)^2 + (a_2\sin\phi)^2$$

$$A^2\cos^2\theta + A^2\sin^2\theta = a_1^2 + 2a_1a_2\cos\phi + a_2^2\cos^2\phi + a_2^2\sin^2\phi$$

$$A^2\left(\cos^2\theta + \sin^2\theta\right) = a_1^2 + 2a_1a_2\cos\phi + a_2^2\left(\cos^2\phi + \sin^2\phi\right)$$

$$A^2 = a_1^2 + 2a_1a_2\cos\phi + a_2^2 \quad \text{... (7)}$$

$$A^2 = a_1^2 + 2a_1a_2\cos\phi + a_2^2 \quad \text{... (8)}$$

Hence the resultant intensity of the combined wave is directly proportional to $\cos\phi$. (1 mark)
I is maximum when $\cos\phi = +1$.

Case (1)

$\cos\phi$ becomes+1 when $\phi = 0, 2\pi, 4\pi, 6\pi.........2n\pi =$ even multiple of π where n is an integer. $n = 0,1,2,3......$

$$I_{\max} = I_1 + 2\sqrt{I_1}\sqrt{I_2} + I_2 = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

$$A^2 = a_1^2 + 2a_1a_2 + a_2^2 \Rightarrow A_{\max}^2 = (a_1 + a_2)^2$$

In this case the two waves are said to undergo **constructive interference**. Since the resultant intensity is maximum, bright fringes are produced. The crest of one wave falls on the crest of the other. The amplitudes get added up.
We know that a phase difference of 2π corresponds to a path difference of λ.
$\therefore$ $2n\pi = n(2\pi) \Rightarrow n\lambda$ where $n = 0,1,2,3...$
Hence the condition for constructive interference is that the path difference between the two waves must be $n\lambda$.

Case (2)

I is minimum when $\cos\phi = -1$. (1 mark)
$\cos\phi$ becomes -1 when $\phi = \pi, 3\pi, 5\pi....... (2n-1)\pi =$ odd multiple of π where n is an integer. $n = 1,2,3....$

$$I_{\max} = I_1 - 2\sqrt{I_1}\sqrt{I_2} + I_2 = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

$$A^2 = a_1^2 - 2a_1a_2 + a_2^2 \Rightarrow A_{\min}^2 = (a_1 - a_2)^2$$

In this case the two waves are said to undergo **destructive interference**. Since the resultant intensity is minimum, dark fringes are produced. The crest of one wave falls on the trough of the other.
We know that a phase difference of π corresponds to a path difference of $\frac{\lambda}{2}$.
Hence the condition for destructive interference is that the path difference between the two waves must be $(2n-1)\frac{\lambda}{2}$. where $n = 1,2,3...$ (1 mark)

$$I_{\min} = I_1 - 2\sqrt{I_1}\sqrt{I_2} + I_2 = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

$$A^2 = a_1^2 - 2a_1a_2 + a_2^2 \Rightarrow A_{\min}^2 = (a_1 - a_2)^2$$

$(5) \div (4)$

$$\tan\theta = \frac{a_2\sin\phi}{a_1 + a_2\cos\phi}$$

Note: $\frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$

Note: *When the two light waves have the same amplitude, then* $a_1 = a_2 = a$

In this case

$A^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos\phi$

$= a^2 + a^2 + 2a^2 \cos\phi = 2a^2 + 2a^2 \cos\phi = 2a^2(1+\cos\phi)$

$= \left(2a^2\right)\left(2\cos^2\frac{\phi}{2}\right)$

$= 4a^2 \cos^2\left(\frac{\phi}{2}\right)$

$\therefore I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$

Now I_0 is the maximum intensity at centre of the fringe systems. I is the intensity at any point where the phase difference between the two superposing waves is ϕ.

11. Monochromatic light of wavelength $589\,\text{nm}$ **is incident from air on a water surface. What are the wavelength, frequency and speed of (a) reflected and (b) refracted light?**
Refractive index of water $= 1.33$

Answer: $\lambda = 589\,\text{nm}$

$c = 3\times10^8\ \text{ms}^{-1}$

$\nu = \frac{c}{\lambda} = \frac{3\times10^8}{589\times10^{-9}}$

$= 5.09\times10^{14}\,\text{Hz}$ (1 mark)

For the reflected ray

Frequency $= 5.09\times10^{14}\,\text{Hz}$

Wavelength $= 589\,\text{nm}$

For the refracted ray

Frequency $= 5.09\times10^{14}\,\text{Hz}$

(When light travels from one medium to another, frequency will not change). (1 mark)

Velocity changes to v such that $v = \frac{c}{n}$

$v = \frac{3\times10^8}{1.33} = 2.25\times10^8\ \text{ms}^{-1}$ (1 mark)

Let $\lambda' =$ wavelength in water

$\frac{\lambda}{\lambda'} = \frac{c}{v} = n$ (1 mark)

$\lambda' = \frac{\lambda}{n} = \frac{589}{1.33} = 444\,\text{nm}$ (1 mark)

12. State Brewster's law. Show that when a ray of light is incident at polarising angle on the surface of a transparent medium, the reflected and refracted rays are perpendicular to each other.

Answer:

Consider a ray of unpolarised light PO to be incident on the surface AB of a glass slab $ABCD$. A part of the ray is reflected along OQ and the remaining part is refracted. Let θ_p be the angle of incidence such that θ_p is the angle of polarisation.

Then the angle of refraction is r_p.

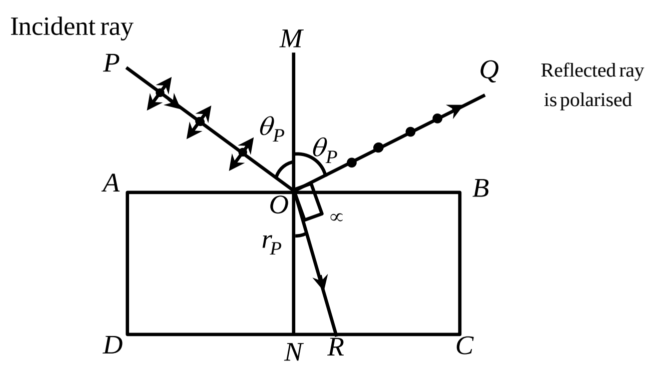


Fig. (1 mark)

Now $n = \tan\theta_p$ …(1) (By Brewster's law) (1 mark)

Where $n =$ refractive index of the medium of which the slab is made.

Also, $n = \dfrac{\sin\theta_p}{\sin r_p}$ …(2) (By Snell's law) (1 mark)

$\therefore \tan\theta_p = \dfrac{\sin\theta_p}{\sin r_p}$

$\dfrac{\sin\theta_p}{\cos\theta_p} = \dfrac{\sin\theta_p}{\sin r_p}$

$\therefore \cos\theta_p = \sin r_p$

$\sin\left(90^\circ - \theta_p\right) = \sin r_p$

$\therefore 90^\circ - \theta_p = r_p$

Or $\theta_p + r_p = 90^\circ$ …(3) (1 mark)

Let α be the angle between the reflected ray (which is fully polarised) and the refracted ray (which is partially polarised).

From the figure,

$\theta_p + \alpha + r_p = 180^\circ$

$\alpha = 180^\circ - \theta_p - r_p$

$= 180^\circ - \left(\theta_p + r_p\right)$

$= 180^\circ - 90^\circ$

$\alpha = 90^\circ$

Hence the reflected ray is perpendicular to the refracted ray when light is polarised by reflection.

(1 mark)