**Blue Print (As per PU Board)**

| Topic | 1 mark questions | 2 marks questions | 3 marks questions | 5 marks questions | Total Marks |
|---|---|---|---|---|---|
| 3 Dimensional Geometry | 1 | 1 | 1 | 1 | 11 |

### One mark questions

1. **Find the direction cosines of the normal to the plane $x+y+z=1$**

Answer: $ax+by+cz=1 \quad \therefore a=1,\ b=1,\ c=1 \qquad \sqrt{a^2+b^2+c^2}=\sqrt{1^2+1^2+1^2}=\sqrt{3}$

$\div(x+y+z=1)$ by $\sqrt{3} \Rightarrow \frac{1}{\sqrt{3}}x+\frac{1}{\sqrt{3}}y+\frac{1}{\sqrt{3}}z=\frac{1}{\sqrt{3}}$. DC's are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

2. **Find the equation of plane with intercept $3$ on the $y$ axis and parallel to $zox$ plane**

Answer: Equation of plane parallel $zox$ plane is $y=k,\ k=y$ intercept

$\therefore k=3 \Rightarrow$ Equation is $y=3$

3. **Find the direction cosines of a line which makes equal angles with the coordinate axes**

Answer: $\alpha=\beta=\gamma \qquad l^2+m^2+n^2=1 \Rightarrow \cos^2\alpha+\cos^2\alpha+\cos^2\alpha=1$

$\therefore 3\cos^2\alpha=1 \qquad \therefore \cos^2\alpha=\frac{1}{3} \qquad \therefore \cos\alpha=\pm\frac{1}{\sqrt{3}}$

The DC's are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

### Two marks questions

4. **Find the vector equation of the line $\frac{x+3}{2}=\frac{y-5}{4}=\frac{z+6}{2}$**

Answer: we have, $\vec{a}=-3\hat{i}+5\hat{j}-6\hat{k},\ \vec{b}=2\hat{i}+4\hat{j}+2\hat{k}$ (1 mark)

$\therefore \vec{r}=\vec{a}+\lambda\vec{b}$

$\vec{r}=\left(-3\hat{i}+5\hat{j}-6\hat{k}\right)+\lambda\left(2\hat{i}+4\hat{j}+2\hat{k}\right)$ (1 mark)

5. **Show that the points $(2, 3, 4), (-1, -2, 1)$ and $(5, 8, 7)$ are collinear**

Answer: We have $A(2, 3, 4)\ B(-1,-2,1)C(5,8,7)$

The direction ratios of line $AB$ are $(-3, -5, -3)$

The direction ratios of line $BC$ are $(6,10,6)$ (1 mark)

The ratio of direction ratios of $AB$ and $BC$ are

$\frac{-3}{6}=-\frac{5}{10}=-\frac{3}{6}\equiv\frac{1}{2}$ in proportion and $B$ is a common point $\therefore A, B, C$ are collinear (1 mark)

6. **Find the values of $p$ so that the lines $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles**

Answer: $\frac{1-x}{3}, \frac{7y-14}{2p}, \frac{z-3}{2} \Rightarrow \frac{x-1}{-3}=\frac{y-2}{\left(\frac{2p}{7}\right)}=\frac{z-3}{2}$

$\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5} \Rightarrow \frac{(x-1)}{\left(\frac{-3p}{7}\right)}=\frac{y-5}{1}=\frac{z-6}{-5}$

$a_1 = -3,\ b_1 = \frac{2p}{7},\ c_1 = 2, \quad a_2 = \frac{-3p}{7},\ b_2 = 1,\ c_2 = -5$ (1 mark)

Condition for perpendicular lines, $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$-3\times\frac{-3p}{7} + \frac{2p}{7}\times 1 + 2\times -5 = 0$

$\frac{9p}{7} + \frac{2p}{7} - 10 = 0$

$11p - 70 = 0 \qquad \Rightarrow p = \frac{70}{11}$ (1 mark)

**Three marks questions**

**7. Find the coordinates of foot of the perpendicular drawn from the origin to the plane** $2x - 3y + 4z - 6 = 0$

Answer: Let the foot of perpendicular be $(x_1, y_1, z_1)$

Direction ratios are $(x_1, y_1, z_1)$

Equation of plane in normal form $\frac{2}{\sqrt{29}}x - \frac{3}{\sqrt{29}}y + \frac{4}{\sqrt{29}}x = \frac{6}{\sqrt{29}} \qquad \ldots(1)$

Where $\frac{2}{\sqrt{29}}, -\frac{3}{\sqrt{29}}$ and $\frac{4}{\sqrt{29}}$ are direction cosines (1 mark)

Since direction ratios and direction cosines are proportional, we have $\frac{x_1}{\left(\frac{2}{\sqrt{29}}\right)} = \frac{y_1}{\left(\frac{-3}{\sqrt{29}}\right)} = \frac{z_1}{\left(\frac{4}{\sqrt{29}}\right)} = k$

$\therefore x_1 = \frac{2}{\sqrt{29}}k,\ y_1 = \frac{-3}{\sqrt{29}}k,\ z_1 = \frac{4}{\sqrt{29}}k$ (1 mark)

Substituting these in $2x - 3y + 4z - 6 = 0$ we get

$k = \frac{6}{\sqrt{29}}$

$\therefore$ The foot of perpendicular is $\left(\frac{12}{29}, \frac{-18}{29}, \frac{24}{29}\right)$ (1 mark)

**8. Find the shortest distance between the lines** $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$

**and** $\vec{r} = -4\hat{i} - \hat{k} + \mu\left(3\hat{i} - 2\hat{j} - 2\hat{k}\right)$

Answer: $\vec{a}_1 = 6\hat{i} + 2\hat{j} + 2\hat{k},\ \vec{b}_1 = \hat{i} - 2\hat{j} + 2\hat{k}$

$\vec{a}_2 = -4\hat{i} - \hat{k},\quad \vec{b}_2 = 3\hat{i} - 2\hat{j} - 3\hat{k}$

$\therefore\ \vec{a}_2 - \vec{a}_1 = -10\hat{i} - 2\hat{j} - 3\hat{k}$ (1 mark)

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} i & j & k \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix} = 8\hat{i} + 8\hat{j} - 4\hat{k}$$

$\therefore \left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{8^2 + 8^2 + 4^2} = \sqrt{144} = 12$ (1 mark)

$$\therefore\ S.D = \frac{\left|\left(\vec{a}_2 - \vec{a}_1\right)\cdot\left(\vec{b}_1 \times \vec{b}_2\right)\right|}{\left|\vec{b}_1 \times \vec{b}_2\right|}$$

$S.D = \frac{|-80 - 16 - 12|}{12} = \frac{108}{12} = 9$ (1 mark)

**9. Find the shortest distance between the lines** $\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$ **and** $\vec{r}=(s+1)\hat{i}+(2s-1)\hat{j}+(2s+1)\hat{k}$

Answer: The given equations can be written as

$\vec{r}=\hat{i}-2\hat{j}+3\hat{k}+t\left(-\hat{i}+\hat{j}-2\hat{k}\right)$

$\vec{r}=\hat{i}-\hat{j}-\hat{k}+s\left(\hat{i}+2\hat{j}-2\hat{k}\right)$

$\vec{a}_1=\hat{i}-2\hat{j}+3\hat{k},\ \vec{b}_1=-\hat{i}+\hat{j}-2\hat{k}$

$\vec{a}_2=\hat{i}-\hat{j}-\hat{k},\ \vec{b}_2=\hat{i}+2\hat{j}-2\hat{k}$

$\vec{a}_2-\vec{a}_1=\hat{j}-4\hat{k}$ (1 mark)

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix} i & j & k \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}=2\hat{i}-4\hat{j}-3\hat{k}$$

$\therefore\left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{4+16+9}=\sqrt{29}$ (1 mark)

$\therefore\ S.D=\dfrac{\left|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}=\dfrac{(-4+12)}{\sqrt{29}}=\dfrac{8}{\sqrt{29}}$ units (1 mark)

## Five marks questions

**10. Derive the equation of a line through a given point and parallel to a given vector $\vec{b}$, both in vector form and Cartesian form**

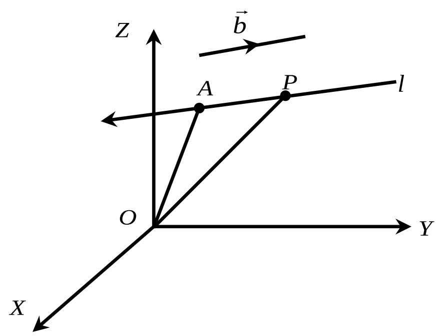


Fig. (1 mark)

Answer: Consider a line $l$, passing through the point $A(x_1, y_1, z_1)$ and parallel to a vector $\vec{b}$.

Let P be any point on $l$ . Coordinates of $P=(x,y,z)$

Let $\overrightarrow{OP}=\vec{r},\ \overrightarrow{OA}=\vec{a}$

$\overrightarrow{AP}$ is parallel to $\vec{b}$ .

$\therefore\overrightarrow{AP}=\lambda\vec{b}.\quad \lambda=$constant (1 mark)

$\overrightarrow{OP}-\overrightarrow{OA}=\lambda\vec{b}$

$\vec{r}-\vec{a}=\lambda\vec{b}$

$\therefore\vec{r}=\vec{a}+\lambda\vec{b}.$ vector equation of the line. (1 mark)

$\vec{r}=x\hat{i}+y\hat{j}+z\hat{k},\ \vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k},\ \vec{b}=a\hat{i}+b\hat{j}+c\hat{k}$

$\vec{r}=a+\lambda b\ \Rightarrow\ x\hat{i}+y\hat{j}+z\hat{k}=\left(x_1\hat{i}+y_1\hat{j}+z_1\hat{k}\right)+\lambda\left(a\hat{i}+b\hat{j}+c\hat{k}\right)$

$x=x_1+\lambda a,\quad y=y_1+\lambda b,\quad z=z_1+\lambda c$ (1 mark)

Eliminating $\lambda$,

$\therefore$ We get $\dfrac{x-x_1}{a}=\dfrac{y-y_1}{b}=\dfrac{z-z_1}{c}$ Cartesian equation. (1 mark)

**Derive the equation of line passing through two given points.**

Answer: Consider a line $l$ passing through two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$.

Let $P(x, y, z)$ be any point on the line.

$\therefore \overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k} = \vec{r}$ Let $\overrightarrow{OB} = \vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ , $\overrightarrow{OA} = \vec{a} = x,\hat{i} + y,\hat{j} + z,\hat{k}$ (1 mark)

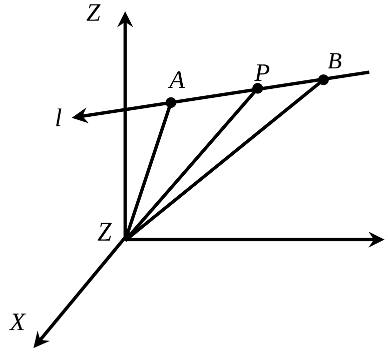


Fig. (1 mark)

The vectors $\overrightarrow{AP}$ and $\overrightarrow{AB}$ are collinear vectors.

$\overrightarrow{AP} = \lambda\overrightarrow{AB}$ (1 mark)

$\overrightarrow{OP} - \overrightarrow{OA} = \lambda\left(\overrightarrow{OB} - \overrightarrow{OA}\right)$

$\vec{r} - \vec{a} = \lambda\left(\vec{b} - \vec{a}\right)$

$\Rightarrow \vec{r} = \vec{a} + \lambda\left(\vec{b} - \vec{a}\right)$ ...(1)

Which is required equation of line in vector form.

Substituting $\vec{r}, \vec{b}$ and $\vec{a}$ in (1)

$x\hat{i} + y\hat{j} + z\hat{k} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} + \lambda\left((x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}\right)$ (1 mark)

$\therefore x = x_1 + \lambda(x_2 - x_1)$

$y = y_1 + \lambda(y_2 - y_1)$

$z = z_1 + \lambda(z_2 - z_1)$

Eliminating $\lambda$, we get

$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$ Cartesian form. (1 mark)

**11. Derive the formula to find shortest distance between parallel lines.**

Answer: consider two parallel lines $l_1$ and $l_2$ with $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ ...(1) and $\vec{r} = \vec{a}_2 + \mu\vec{b}$ ...(2).

$l_1$ and $l_2$ passes through the points $A$ and $B$ with position vectors $\vec{a}_1$ and $\vec{a}_2$ both are parallel to $\vec{b}$.

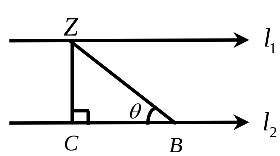


Fig. (1 mark)

Draw a perpendicular $AC$ from the point $A$ to $(l_2)$.

Let $\theta$ be the angle between the line (2) and $AB$. (1 mark)

$\sin\theta = \frac{AC}{AB} \Rightarrow AC = AB\sin\theta$

$\left|\overrightarrow{AB} \times \vec{b}\right| = \left|\overrightarrow{AB}\right|\left|\vec{b}\right|\sin\theta$

$= \left(\left|\overrightarrow{AB}\right|\sin\theta\right)\left|\vec{b}\right|$ (1 mark)

$= \left|\overrightarrow{AC}\right|\left|\vec{b}\right|$ from (3)

$\therefore \left|\overrightarrow{AC}\right| = \dfrac{\left|\overrightarrow{AB} \times \vec{b}\right|}{\left|\vec{b}\right|}$ (1 mark)

$\therefore \text{Shortest Distance} = \left|\overrightarrow{AC}\right| = \dfrac{\left|\left(\vec{a}_2 - \vec{a}_1\right) \times \vec{b}\right|}{\left|\vec{b}\right|}$ (1 mark)