Blue Print (As per PU Board)

Topic	1 mark questions	2 marks questions	3 marks questions	6 marks questions	Total Marks
Linear Programming	1	–	–	1	7

One mark questions

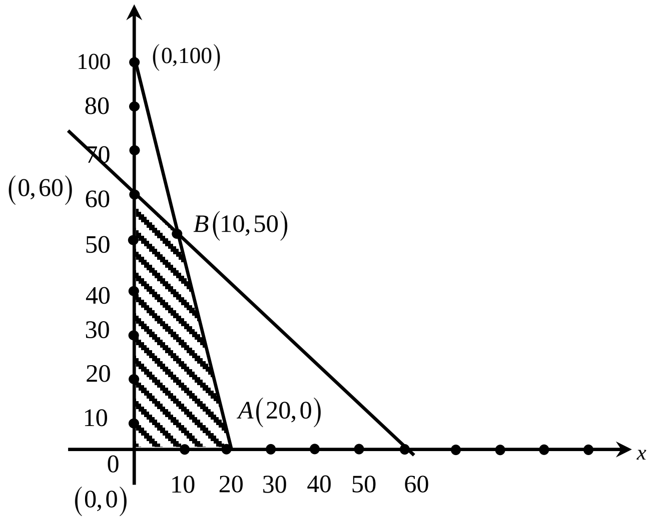
1. **Define Linear Programming Problem.**
   Answer: It deals with the maximization or minimization of a linear function of a number of variables subject to number of conditions on the variables in the form of linear inequations.
2. **Define an objective function.**
   Answer: It is a function of certain variables, which is to be maximized or minimized, subject to given conditions on the variables.
3. **Define constraints.**
   Answer: The conditions on the variable of an objective function are called constraints of a linear programming problem which are in the form of linear inequations.
4. **What are decision variables?**
   Answer: The non-negative variables of a linear programming problem are called decision variables.
5. **Define feasible region.**
   Answer: The region which is common to all constraints of a linear programming problem is called feasible region.
6. **Define feasible solution.**
   Answer: The set of variables $(x \text{ and } y)$ which satisfies all the constraints of a L.P.P. is called feasible solution.
   OR
   Every point in the feasible region of a L.P.P. is called a feasible solution.

Six marks questions

7. **Maximize** $z = 50x + 15y$ **subject to constraints:** $5x + y \le 100, x + y \le 60, x, y \ge 0$
   Answer: The objective function is $z = 50x + 15y$
   Constraints are $5x + y \le 100, x + y \le 60, x, y \ge 0$
   Consider $5x + y = 100$
   Put $y = 0, x = 20$. The point is $(20, 0)$
   $x = 0, y = 100$. The point is $(0, 100)$ (1 mark)
   $(0, 0)$ lies in the half plane of $5x + y \le 100$ ($\therefore 0 + 0 < 100$ which is true)
   Consider $x + y = 60$
   Put $x = 0, y = 60$ and $y = 0, x = 60$

$\therefore$ The points are $(60,0)$ and $(0,60)$. $(0,0)$ lies in the half plane of $x+y\le 60$.

(1 mark)



The shaded bounded region $OABC$ is the feasible region of the given L.P.P. (1 mark)

The corner points are $O(0,0), A(20,0), B(10,50)$ and $C(0,60)$ (1 mark)

Corner points	$z=50x+15y$
$O(0,0)$	0
$A(20,0)$	1,000
$B(10,50)$	1,250
$C(0,60)$	900

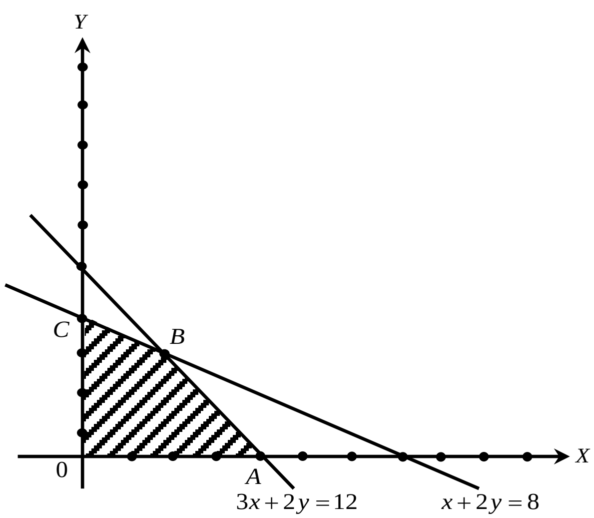
(1 mark)

$\therefore$ Maximum value of $z=1,250$ when $x=10$ and $y=50$ (1 mark)

8. Maximize, $z=-3x+4y$ **subject to** $x+2y\le 8, 3x+2y\le 12, x\ge 0, y\ge 0$

Answer: $z=-3x+4y$ Changing the inequations $x+2y\le 8, 3x+2y\le 12$ into equations.

i.e., $x+2y=8$ ...(1) $3x+2y=12$ ...(2)



(1 mark)

$x+2y=8$

when $\left.\begin{array}{l} y=0, x=8 \\ x=0, y=4 \end{array}\right\}$ the points are $(8,0)$ and $(0,4)$

$3x+2y=12$

Put $\left.\begin{array}{l} x=0, y=6 \\ y=0, x=4 \end{array}\right\}$ The points are $(4,0)$ and $(0,6)$ (1 mark)

Solving (1) & (2) we get point of intersection of these two lines as $(2,3)$.

The feasible region is the shaded region $OABC$. (1 mark)

The corner points of this region are $O(0,0)A(4,0), B(2,3)C(0,6)$. (1 mark)

The value of Z at these points are

Corner points	$z=-3x+4y$
$O(0,0)$	0
$A(4,0)$	–12
$B(2,3)$	6
$C(0,4)$	16

(1 mark)

$z_{\max}=16$ at $(0,4)$. $z_{\min}=-12$ at $(4,0)$ (1 mark)

9. Minimize: $z=x+2y$, **subject to** $2x+y\geq 3, x+2y\geq 6, x, y\geq 0$

Answer: Minimize $z=x+2y$

Changing the in equations $2x+y\geq 3, x+2y\geq 6$ into equations, $2x+y=3$ and $x+2y=6$ and solving these two equations we get point of intersection at $(0,3)$

Consider $2x+y=3$

Put $\left.\begin{array}{l} y=0, x=3/2 \\ x=0, y=3 \end{array}\right\}$ the points are $\left(\frac{3}{2},0\right)$ and $(0,3)$

Consider $x+2y=6$

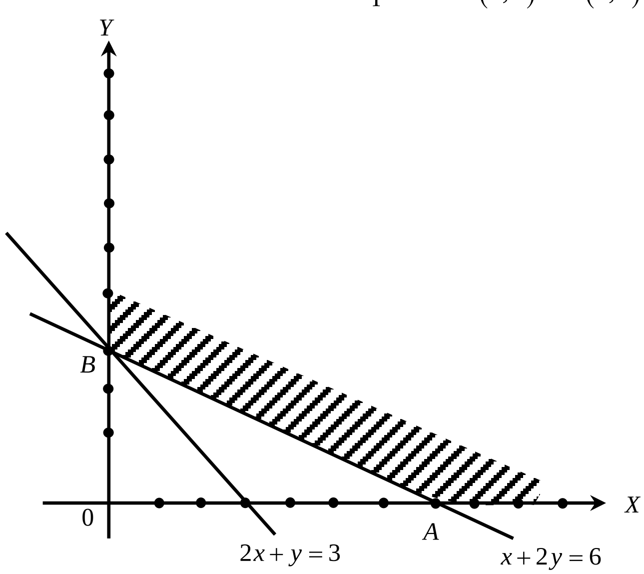
Put $\left.\begin{array}{l} y=0, x=6 \\ x=0, y=3 \end{array}\right\}$ the points are $(6,0)$ and $(0,3)$ (1 mark)

The feasible region is the shaded region which is unbounded. (1 mark)

The corner points of the region are $A(6,0)$ and $B(0,3)$ (1 mark)

At $\left.\begin{array}{ll} A(6,0) & z=6 \\ B(0,3) & z=6 \end{array}\right\}$ (1 mark)

The minimum value occurs at two points $A(6,0)\&B(0,3)$.



(1 mark)

$\therefore$ L.P.P. has infinitely many solutions, attained at every point on the line joining the points A and B i.e., $x+2y=6$. The region $z<6$ i.e., $x+2y<6$ has no common point in the feasible region. (1 mark)

10. Maximize and Minimize: $z=5x+10y$ **subject to** $x+2y\le 120, 3x+y\ge 60, x-2y\ge 0, x, y\ge 0$.

Answer: Maximize and Minimize $z=5x+10y$, changing the in equations $3x+y\ge 60, x-2y\ge 0$ and $x+2y\le 120$ into equations, we get $3x+y=60, x-2y=0, x+2y=120$.

Consider $3x+y=60$

Put $\left.\begin{matrix} x=0, y=60 \\ y=0, x=20 \end{matrix}\right\}$ the points are $(20,0)$ and $(0,60)$

The line AB represents $3x+y=60$

Consider $x-2y=0$

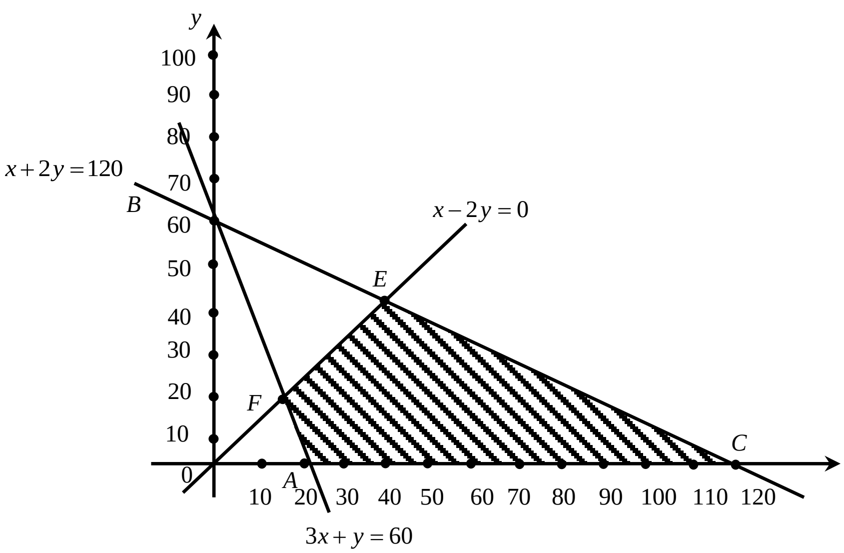
Put $x=0, y=0$

OE represents the line $x-2y=0$

Consider $x+2y=120$

Put $x=0, y=60, y=0, x=120$

The points are $(120,0),(0,60)$ (1 mark)



(1 mark)

The line CB represents $x+2y=120$.

E is point of $x-2y=0$ and $x+2y=120$

F is point of intersection of $3x+y=60$ and $x+2y=0$

$E(60,30)$

$F\left(\frac{120}{7},\frac{60}{7}\right)$

The feasible region is the shaded region $ACEF$ (1 mark)

The corner points are, $A(20,0)C(120,0)E(60,30)$ and $F\left(\frac{120}{7},\frac{60}{7}\right)$ (1 mark)

Corner points	$z=5x+10y$
$A(20,0)$	100
$C(120,0)$	600
$E(60,30)$	600

$F\left(\frac{120}{7},\frac{60}{7}\right)$	$\frac{1200}{7}$

(1 mark)

$Z_{\text{maximum}} = 600$ at $C(120,0)$ and $E(60,30)$.

The maximum value attains at every point on the line CE.

$Z_{\text{minimum}} = 100$ at $(20,0)$ (1 mark)

11. Minimize and maximize $z = x+2y$ subject to $x+2y \geq 100, 2x-y \leq 0, 2x+y \leq 200, x, y \geq 0$.

Answer: Changing the inequations $x+2y \geq 100, 2x-y \leq 0$ and $2x+y \leq 200$ into equations as $x+2y=100, 2x-y=0, 2x+y=200$. Consider $x+2y=100$

Put $\left.\begin{matrix} x=0, y=50 \\ y=0, x=100 \end{matrix}\right\}$ The points are $A(100,0)$ and $B(0,50)$ (1 mark)

The line AB represents the line $x+2y=100$.

Consider $2x-y=0$ $x=0, y=0$. The line OE represents $2x-y=0$

Consider $2x+y=200$

Put $\left.\begin{matrix} x=0, y=200 \\ y=0, x=100 \end{matrix}\right\}$ The points are $A(100,0) C(0,200)$

The line AC represents the line $2x+y=200$

D is point of intersection of $x+2y=100$ and $2x-y=0$. Which is $(20,40)$ (1 mark)

$E(50,100)$ is point of intersection of $2x+y=200$ and $2x-y=0$.

The feasible region is the shaded region $BDEC$.

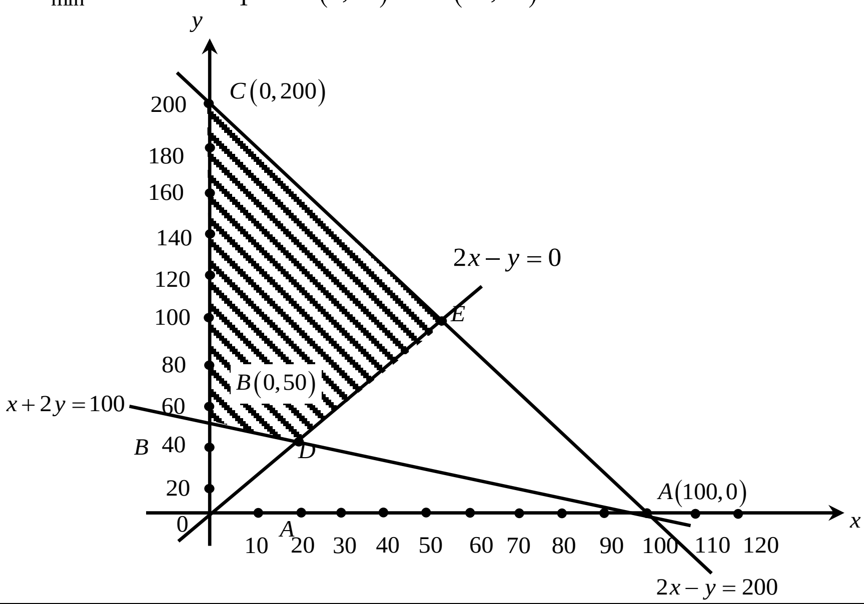
The corner points are $B(0,50), D(20,40) E(50,100)$ and $C(0,200)$ (1 mark)

Corner points	$z=x+2y$
$B(0,50)$	100
$D(20,40)$	100
$E(50,100)$	250
$C(0,200)$	400

(1 mark)

$z_{\max} = 400$ at $(0,200)$

$z_{\min} = 100$ at two points $(0,50)$ and $(20,40)$.



(1 mark)

Hence z_{minimum} at all points joining the line BD. (1 mark)

12. There are two factories located one at place P and the other at place Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively $5, 5$ and 4 units of the commodity while the production capacity of the factories at P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below.

From/To	Cost (in Rs)		
	A	B	C
P	160	100	150
Q	100	120	100

How many units should be transported from each factory in order that the transportation cost is minimum. What will be the minimum transportation cost?

Answer: Let x units and y units of the commodity be transported from the factory at P to the depots at A and B respectively.

$\therefore (8-x-y)$ units will be transported to depot at C

We have $x \geq 0, y \geq 0, 8-x-y \geq 0, x+y \leq 8$

A requires 5 units. Since x units are transported from the factory at P, the reaming $(5-x)$ need to be transported from Q.

$\therefore 5-x \geq 0 \Rightarrow x \leq 5$

Similarly $(5-y)$ and $6-(5-x+5-y) = x+y-4$ units are to be transported from Q to B and C respectively.

$\therefore 5-y \geq 0, x+y-4 \geq 0$

i.e., $y \leq 5, x+y \geq 4$ (1 mark)

Total transportation cost $Z = 160x + 100y + 100(5-x) + 120(5-y) + 100(x+y-4) + 150(8-x-y)$

$= 10(x - 7y + 190)$

Minimize $Z = 10(x-7y+190)$ subject to constraints

$x \geq 0, y \geq 0, x+y \leq 8, x \leq 5, y \leq 5, x+y \geq 4$ $x = 5, y = 5$

Consider $x + y = 8$

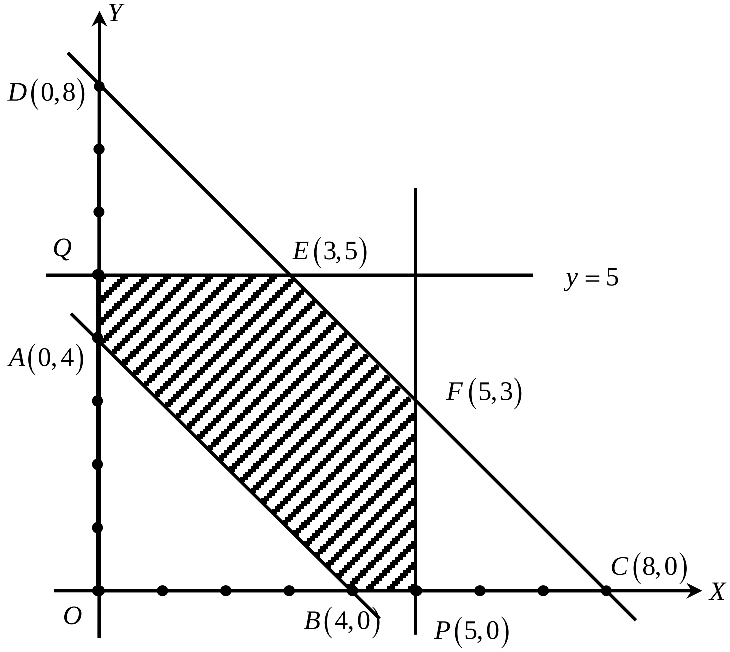
$x = 0, y = 8$

$y = 0, x = 8$

$x + y = 4$

$x = 0, y = 4$ (1 mark)

$y = 0, x = 4$

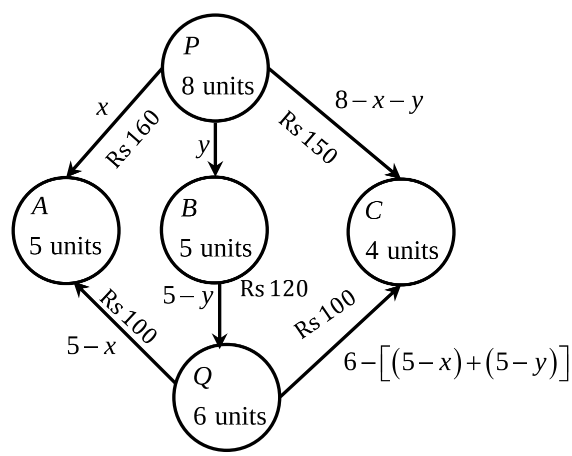


(1 mark)

The feasible region is bounded which is shaded. $(AQEFPB)$

The corner points are $(0,4),(0,5),(3,5),(5,3),(5,0)$ and $(4,0)$ (1 mark)

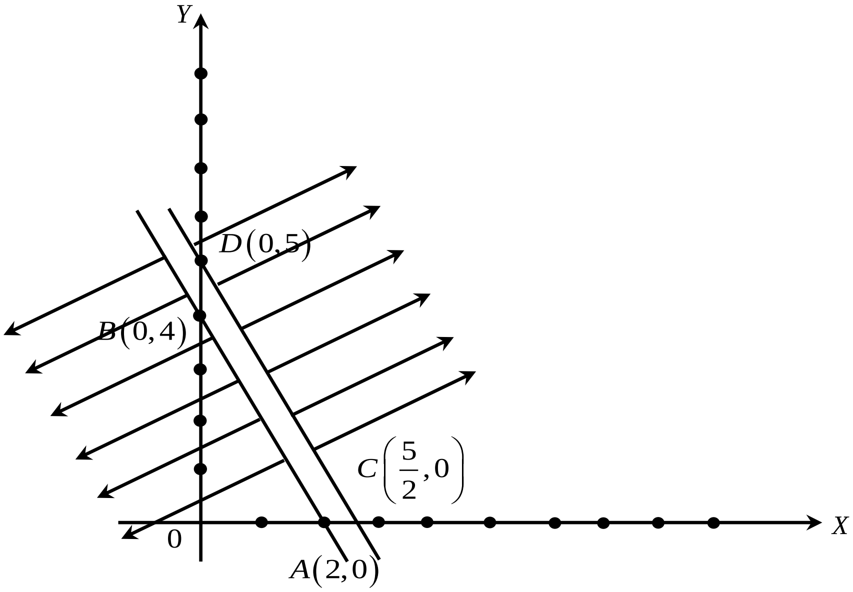
Corner points	$Z = 10(x - 7y + 190)$
$(0,4)$	1620
$(0,5)$	1550
$(5,3)$	1740
$(3,5)$	1580
$(5,0)$	1950
$(4,0)$	1940



(1 mark)

Minimum value of Z is 1550 at $(0,5)$

The transportation cost is minimum. Factory at P to deliver $0,5$ and 3 units and Factory at Q , $5,0,1$ to depots A, B and C . (1 mark)



(1 mark)

The line CD represents $4x+2y=10$.

The lines AB and CD are parallel to each other. Thus there is no common region between $2x+y\le 4$ and $4x+2y\ge 10$. (1 mark)

Hence there is no feasible region and hence no maximum value for z. (2 marks)