Blue Print (As per PU Board)

Topic	1 mark questions	2 marks questions	3 marks questions	5 marks questions	Total Marks
Inverse Trigonometric Functions	1	1	1	1	11

One mark questions

1. Write the principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

Ans: $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)=\pi-\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$ $\left[\cot^{-1}(-x)=\pi-\cot^{-1}x\right]$

2. Write the set of values of x **for which** $2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ **holds**

Ans: The set of values of x is $|x|<1$

3. If $\tan^{-1}x=\frac{\pi}{10}$ **for some** $x\in R$**, then the value of** $\cot^{-1}x$ **is**

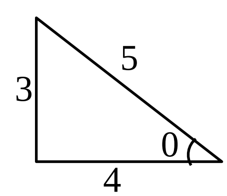
Ans: By using $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$

$\frac{\pi}{10}+\cot^{-1}x=\frac{\pi}{2}$

$\cot^{-1}x=\frac{\pi}{2}-\frac{\pi}{10}=\frac{5\pi-\pi}{10}=\frac{4\pi}{10}$

4. Solve $\sin^{-1}x=\tan^{-1}\frac{3}{4}$

Ans: Let $\tan^{-1}\frac{3}{4}=\theta\tan\theta=\frac{3}{4}$



$\therefore\sin\theta=\frac{3}{5}\Rightarrow\theta=\sin^{-1}\frac{3}{5}$

$\sin^{-1}=x\sin^{-1}\frac{3}{5}\Rightarrow x=\sin\sin^{-1}\frac{3}{5}$ [By using $\sin\sin^{-1}x=x$]

$\therefore x=\frac{3}{5}$

Two marks questions

5. Evaluate $\sin\left\{\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right\}$

Ans: $\sin\left\{\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right\}=\sin\left\{\frac{\pi}{3}+\sin^{-1}\frac{1}{2}\right\}=\sin\left\{\frac{\pi}{3}+\sin^{-1}\frac{1}{2}\right\}=\sin\frac{3\pi}{6}=\sin\frac{\pi}{2}=1$

6. Find the simplest form of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$

Ans: $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$ [By using $\sec^{-1}(-x)=\pi-\sec^{-1}x$]

$\tan^{-1}\sqrt{3}-\left(\pi-\sec^{-1}2\right)$

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$$\frac{\pi}{3}-\left(\pi-\frac{\pi}{3}\right)=\frac{\pi}{3}-\frac{2\pi}{3}=-\frac{\pi}{3}$$

7. Prove that $2\sin^{-1}x=\sin^{-1}\left(2x\sqrt{1-x^2}\right)$, $\frac{-1}{\sqrt{2}}\le x\ge\frac{1}{\sqrt{2}}$

Ans: Let $\sin^{-1}x=\theta\Rightarrow x=\sin\theta$

$\cos\theta=\sqrt{1-\sin^2\theta}=\sqrt{1-x^2}$

Consider $\sin 2\theta=2\sin\theta\cos\theta\Rightarrow\sin 2\theta=2x\sqrt{1-x^2}$

$=2\theta=\sin^{-1}2x\sqrt{1-x^2}$ $=2\sin^{-1}x=\sin^{1}\left(2x\sqrt{1-x^2}\right)$

8. Evaluate $\tan^{-1}\left[\tan\left(\frac{9\pi}{8}\right)\right]$

Ans: Consider $\tan^{-1}\tan\frac{9\pi}{8}=\tan^{-1}\tan\left(\pi+\frac{\pi}{8}\right)=\tan^{-1}\left(-\tan\frac{\pi}{8}\right)$

$=\tan^{-1}\tan\left(-\frac{\pi}{8}\right)=\frac{\pi}{8}$ [By using $\tan^{-1}\tan\theta=\theta$]

Three marks questions

9. Prove that $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ **when** $xy<1$

Ans: Let $\tan^{-1}x=A\Rightarrow x=\tan A$ & $\tan^{-1}y=B\Rightarrow y=\tan B$

Consider $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$ $\left[\tan(A+B)=\frac{x+y}{1-xy}\right]$

$A+B=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$\tan^{-1}x+\tan^{-1}y=\tan^{1}\left(\frac{x+y}{1-xy}\right)$

10. Prove that $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$

Ans: Consider $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}$

By using $2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}$

$$\tan^{-1}\left(\frac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2}\right)+\tan^{-1}\frac{1}{7}\Rightarrow\tan^{-1}\left(\frac{\frac{1}{2}\times 2}{\frac{4-1}{4}}\right)+\tan^{-1}\frac{1}{7}$$

$$\Rightarrow\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{1}{7}\Rightarrow\tan^{-1}\left(\frac{\frac{28+3}{21}}{\frac{21-4}{21}}\right)$$

$\Rightarrow\tan^{-1}\left(\frac{31}{17}\right)$ [By using $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$]

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11. If $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$ **then find the value of** x

Ans: Consider $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}\Rightarrow \tan^{-1}\left(\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right)=\frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{(x-1)(x+2)+(x+1)(x-2)}{(x-2)(x+2)}}{\frac{(x-2)(x+2)-(x-1)(x+1)}{(x-2)(x+2)}}\right]=\frac{\pi}{4}\Rightarrow \frac{x^2-x+2x-2+x^2+x-2x-2}{(x^2-4)-(x^2-1)}=\tan\frac{\pi}{4}$$

$$\Rightarrow \frac{2x^2-4}{x^2-4-x^2+1}=1\Rightarrow \frac{2x^2-4}{-3}=1\Rightarrow 2x^2-4=-3\Rightarrow 2x^2=4-3\Rightarrow x^2=\frac{1}{2}\Rightarrow x=\pm\frac{1}{\sqrt{2}}$$

12. Prove that $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\sin^{-1}\frac{1}{\sqrt{5}}$

Ans: Consider $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}$

$$\tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\left(\frac{2}{9}\right)}\right)=\tan^{-1}\left(\frac{\frac{9+8}{36}}{\frac{36-2}{36}}\right)=\tan^{-1}\left(\frac{17}{34}\right)=\tan^{-1}\frac{1}{2}$$

$$\tan^{-1}\frac{1}{2}=\sin^{-1}\left[\frac{1/2}{\sqrt{1+(1/2)^2}}\right]$$ [By using $\tan^{-1}x=\sin^{-1}\frac{x}{\sqrt{1+x^2}}$]

$$=\sin^{-1}\frac{1/2}{\sqrt{5/4}}=\sin^{-1}\frac{1/2}{\sqrt{5}/2}=\sin^{-1}\frac{1}{\sqrt{5}}$$

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