Blue Print (As per PU Board)

Topic	1 mark questions	2 marks questions	3 marks questions	5 marks questions	Total Marks
Relations & Functions	1	1	1	1	11

One mark questions

1. **Give an example of a relation which is symmetric and transitive but not Reflexive**

Ans: Let $A=\{1,2,3\}$

$R=\{(1,2)(2,1)(1,1)\}$

$(1,2)\in R \Rightarrow (2,1)\in R$ $\therefore$ It is symmetric

$(1,2)\&(2,1)\in R \Rightarrow (1,1)\in R$ $\therefore$ It is transitive

$(1,1)\in R$ but $(2,2)\notin R$ $\therefore$ It is not reflexive

2. **Let * be a Binary operation defined on set of rational number by $a*b=\frac{ab}{4}$. Find the identity element**

Ans: By Identify law

$a*e=e*a=a$

Consider $a*e=a$

$\frac{ae}{4}=a$

$e=\frac{4a}{a}=4$ $e=4\in Q$

3. **On the set of $Q*$ is defined by $a*b=a-b\ \forall\ a,b\in Q$ Is $*a$ B.O. Justify your answer**

Ans: $A=\{1,2,3\}$

$a,b\in Q$ Let $a=\frac{1}{2}$ $b=\frac{2}{3}$

$a*b=a-b=\frac{1}{2}-\frac{2}{3}=\frac{3-4}{6}=-\frac{1}{3}\in Q$

$\therefore *$ is Binary Operation

Two marks questions

4. **Define a binary operation on a set. Verify whether the operation $*$ defined on z by $a*b=ab+1$ is binary operation or not**

Ans: Definition: A binary operation $*$ on set A is a function

$*:A\times A\to A$. We denote $*(a,b)$ by $a*b$

On the set of integers $a*b=ab+1$

Let $a=-3,\ b=2$, then $-3*2=-3(2)+1=-6+1=-5\in Z$

Therefore, $*$ is a binary operation

5. **A relation R is defined on the set $A=\{1,2,3,5,6\}$ by $R=\{(x,y): y\text{ is divisible by } x\}$. Verify whether R is symmetric and reflexive or not. Give reason**

Ans: $R=\{(1,1)(1,2)(1,3)(1,5)(1,6),(2,2),(2,6)(3,6)(3,3)(5,5)(5,6)\}$

$(1,1),(2,2),(3,3)(5,5)(6,6)\in R$

Therefore R is reflective

$(1,2)\in R$ but $(2,1)\notin R$

$(1,3)\in R$ but $(3,1)\notin R$

Therefore R is not symmetric

6. If the mapping f and g are given by $f=\{(1,2)(3,5)(4,1)\}$ and $g=\{(2,3)(5,1)(1,3)\}$ write fog

Ans: Domain of $f=\{1,34\}$ Range of $f=\{2,5,1\}$

Domain of $g=\{2,5,1\}$ Range of $g=\{3,1\}$

$fog(1)=f(3)=5$

$fog(5)=f[g(5)]=f(1)=2$

$fog(2)=f[g(2)]=f(3)=5$

Three marks questions

7. Verify whether the function $f:A\to B$ where $A=R-\{3\}$ and $B=R-\{1\}$ defined by $f(x)=\dfrac{x-2}{x-3}$ is one-one and onto or not given reasons

Ans: Let $x_1,x_2\in R-\{3\}\Rightarrow f(x_1)=f(x_2)$

$\dfrac{x_1-2}{x_1-3}=\dfrac{x_2-2}{x_2-3}$

$(x_1-2)(x_2-3)=(x_2-2)(x_1-3)$

$x_1x_2-2x_2-3x_2+6=x_1x_2-2x_1-3x_2+6$

$-3x_1+2x_1=-3x_2+2x_2$

$-x_1=-x_2$

$x_1=x_2$ $\therefore f$ is one-one

$\forall x\in R-\{3\}$ $\exists\, y$ such that $f(x)=y\Rightarrow\dfrac{x-2}{x-3}=y$

$x-2=y(x-3)\Rightarrow x-2=xy-3y\Rightarrow 3y-2=xy-x\Rightarrow 3y-2=x(y-1)$

$x=\dfrac{3y-2}{y-1}\in R-\{1\}$

$$f(x)=y\Rightarrow f\left[\frac{3y-2}{y-1}\right]=\frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3}\,\frac{3y-2-2y+2}{3y-2-3y+3}=\frac{y}{1}=y$$

$\therefore f(x)=y$ $\therefore f$ is onto

8. Show that the Relation R is the set of all integers defined by $R=(a,b):2$ divides $a-b\}$ is an equivalence relation

Ans: Reflexive: $\forall\, a\in R(a,a)\in R\Rightarrow a-a$ is divisible by $2\Rightarrow 0$ is divisible by 2

$\therefore R$ is reflexive

Symmetric: $\forall\, a,b\in R$ of $(a,b)\in R\Rightarrow(b,a)\in R$

$(a,b)\in R\Rightarrow a-b$ is divisible by 2

$-(b-a)$ is divisible by 2

$\therefore(a,b)\in R\Rightarrow(b,a)\in R$

$\therefore R$ is symmetric

Transitive: $\forall\, a,b,c\in R$ if $(a,b)\&(b,c)\in R\Rightarrow(a,c)\in R$

$(a,b)\in R\Rightarrow a-b$ is divisible by 2

$(b-c)\in R\Rightarrow b-c$ is divisible by $2\Rightarrow a-b+b-c=a-c$ is divisible by $2\Rightarrow(a,c)\in R$

$\therefore R$ is transitive

$\therefore R$ is an equivalence Relation

9. Show that the Relation R in the set $A=\{1,2,3,4,5\}$ given by $R=\{(a,b):|a-b| \text{ is even}\}$ is an equivalence relation

Ans: Reflexive: $(a,a)\Rightarrow|a-a|=0$ is even $\in R$

$\therefore (a,a)\in R$ $\quad\therefore R$ is reflexive

Symmetric: $(a,b)\in R\Rightarrow|a-b|$ is even

$\Rightarrow|-b+a|$ is even $\Rightarrow|-(b-a)|$ is even $\Rightarrow|b-a|$ is even $\Rightarrow(b,a)\in R$

$\therefore R$ is symmetric

Transitive: $(a,b)\in R\Rightarrow|a-b|$ is even

$(b,c)\in R\Rightarrow|b-c|$ is even, then $|a-b+b-c|$ is even

$\Rightarrow|a-c|$ is even $\Rightarrow(a,c)\in R$

$\therefore R$ is transitive $\quad\therefore R$ is an equivalence relation

Five marks questions

10. Verify the function $f:N\to Y$ defined by $f(x)=4x+3$ where $Y=\{y:y=4x+3x\in N\}$ is invertible or not, write inverse of $f(x)$ if exists

Ans: Consider an arbitrary element y of Y

By definition $Y\Rightarrow y=4x+3$ for some x in the domain N

This shows that $x=\dfrac{y-3}{4}$

Now define $g:Y\to N$ by $g(y)=\dfrac{y-3}{4}$

$$fog(y)=f[g(y)]=f\left(\frac{(y-3)}{4}\right)=\frac{4(y-3)}{4}+3=y-3+3=y$$

This shows that $gof=I_N$ and $fog=I_Y$ which implies that f is invertible and g is the inverse of f

11. Let $f:N\to R$ defined by $f(x)=4x^2+12x+15$ show that $f:N\to S$ where S is the range of the function is invertible and also find the inverse

Ans: Let y be an arbitrary element of range f. Then $4x^2+12x+5$, for some x in N

$\Rightarrow 4x^2+12x+15=y\Rightarrow(2x)^2+2\cdot 2x(3)+15=y$

$\Rightarrow(2x)^2+12x+9+15-9=y\Rightarrow(2x+3)^2+6=y$

$\therefore (2x+3)^2=y-6$

$2x+3=\sqrt{y-6}\Rightarrow 2x=\sqrt{y-6}-3\Rightarrow x=\dfrac{\sqrt{y-6}-3}{2}$ as $y\ge 6$

Let us define $g:S\to N$ by $g(y)=\dfrac{\sqrt{y-6}-3}{2}$

$$gof(x)=g[f(x)]=g(4x^2+12x+15)=g\left[(2x+3)^2+6\right]$$

$$=\frac{\sqrt{(2x+3)^2+6-6}-3}{2}=\frac{2x+3-3}{2}=\frac{2x}{2}=x$$

$$fog(x) = f[g(x)] = f\left(\frac{\sqrt{y-6}-3}{2}\right) = \left[f\left(\frac{2\sqrt{y-6}-3}{2}\right)+3\right]^2 + 6$$

$\therefore$ $gof = I_N$ and $fog = I_S$. This implies f is invertible with $f^{-1} = g$

$$f^{-1} = \frac{\sqrt{y-6}-3}{2}$$

12. Consider $f : R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with

$$f^{-1}(y) = \frac{\left(\sqrt{y+6}\right)-1}{3}$$

Ans: $\forall$ $x_1, x_2 \in R_+$

$f(x_1) = f(x_2) \Rightarrow 9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5 \Rightarrow 9x_1^2 - 9x_2^2 + 6x_1 - 6x_2 = 0$

$9(x_1 + x_2)(x_1 - x_2) + 6(x_1 - x_2) = 0$

$(x_1 - x_2)[9(x_1 + x_2) + 6] = 0 \Rightarrow x_1 = x_2$

$\therefore$ f is one-one

$\forall x \in R_+ \exists y$ such that $f(x) = y$

$9x^2 + 6x - 5 = y \Rightarrow (3x)^2 + 2(3x) + 1 - 5 - 1 = y \Rightarrow (3x+1)^2 - 6 = y \Rightarrow (3x+1)^2 = y + 6$

$3x + 1 = \sqrt{y+6} \Rightarrow 3x = \sqrt{y+6} - 1$

$x = \dfrac{\sqrt{y+6}-1}{3}$ where $y \in [-5, \infty]$

$$f(x) = f\left(\frac{\sqrt{y+6}-1}{3}\right) = 9\left[\frac{\sqrt{y+6}-1}{3}\right]^2 + 6^2\left[\frac{\sqrt{y+6}-1}{3}\right] - 5$$

$$= \frac{9}{9}\left[\left(\sqrt{y+6}\right)^2 + 1 - 2\sqrt{y+6}\right] + 2\left[\sqrt{y+6} - 1\right] - 5$$

$$= y + 6 + 1 - 2\sqrt{y+6} + 2\sqrt{y+6} - 2 - 5$$

$$= y + 6 - 6 = y$$

$\therefore f(x) = y$ $\qquad$ $\therefore f$ is onto $\qquad$ $\therefore f$ is bijective $\quad$ $\therefore f(x) = y$

$x = f^{-1}(y)$ $\qquad$ $\therefore f^{-1}(y) = \dfrac{\sqrt{y+6}-1}{3}$